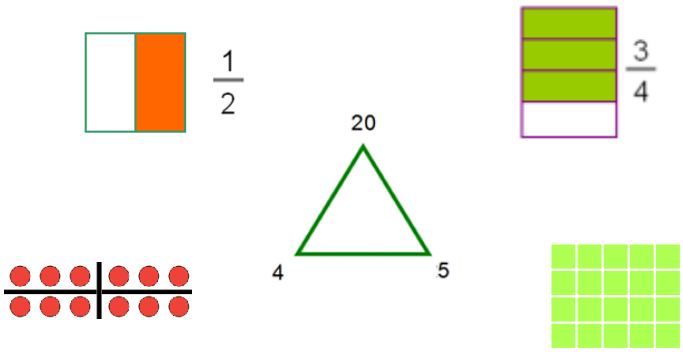
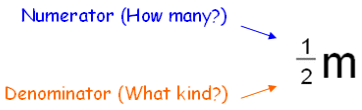
Success with Fractions

For Teachers and Parents!



Rob Porteous

Part 1 Version 2: December 2016



Contents

Subsequent steps will be published as Part 2 to this guide.

Introduction

Partnership in Learning

A child's learning is a partnership between school and home.

So this guide is written for both teachers **and** parents!

The activities are both practical and tablet-based. All have been thoroughly tested in the classroom, and all can be used effectively both in school and at home.

Skills Progression

To understand Maths properly, children need to learn step-by-step with regular opportunities for reinforcement.

Use the following guides, in order. Your children will then be following a carefully planned skills progression. Regularly revising these activities, both in school and at home, will give them a really secure foundation in their core number skills.

Tips for Effective Learning

Many children have difficulty with fractions. There are several things you can do to help your children learn more effectively. The most important are

- Work step by step
- Use practical equipment
- Think and talk about learning
- Regularly reinforce what you have learned

For a comprehensive guide to all of these, see the extra sections at the back!

Fractions: Step by Step

On each page of this guide, you will find 'learning intentions', suggestions for practical activities & discussion and, where appropriate, follow-up activities for the tablet.

To help with planning your children's learning the activities are divided into steps. The activities on one step might normally be covered during the course of a year, with step 1 roughly corresponding to the first year of formal schooling, step 2 to the second etc. However, children develop at different rates. Start at a level where your child(ren) feel(s) secure. Take things steadily, and repeat things regularly to ensure understanding.

As children learn, they need lots of practice to make sure they are really secure with each stage before moving on to the next.

The Maths Investigations tablet activities are designed exactly for this purpose. Activities are referenced in what follows both by 'Step' on the Learning Ladder and by 'Owl' as follows:

	MI Activity	Step	Related Owl
	Finding Half of Even Numbers	2	Divvy
	Quartering Numbers in the 4x-table	3	Quarty

In the examples above, the activity 'Finding Half of Even Numbers' can be found by going to step 2 of the Learning Ladder and looking in the Fractions section under 'Finding Fractions of Things' for an activity with a carrot. It can also be found via the 'Owl' called 'Divvy'.

You will find full guidance on using the tablet activities in the Teacher or Parent Guide on the Maths Investigations web site (www.mathsinvestigations.com).

Step 1

Halves, Halving and Doubling

We begin our exploration of fractions with the everyday ideas of **halving and doubling**. First explore **halves of objects**. Give as much practical experience as possible of **making** halves, and **playing** with halves. Next explore **halves and doubles of groups** of things. Work big with hoops and bean bags at first. As confidence develops, you can graduate to smaller things, like counters.

Learning Intentions

Understand what halves are.

Understand the terms 'doubling' and 'halving'.

Double and half groups of things.

Halving Objects

Refer to halves in everyday experience.

Halve an apple. Discuss: two halves make one whole.

Split mashed potato in half!

Words

Half
Double
Divide
Share
Group

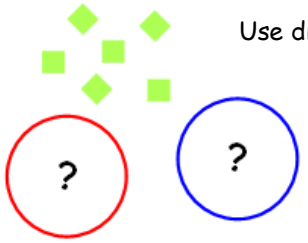
Challenge!

Use 2 hoops and some bean bags. How could you share 6 bean bags fairly between the two hoops?

Discuss double 3 is 6 – half of 6 is 3, etc.

Try with other numbers.

Use different things to share.

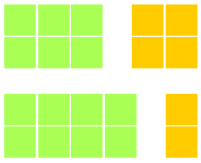


Maths Rap!

Double 3 is 6.
so…
Half of 6 is 3.

Odds and Evens

Put an even number of counters in two rows and they pair up.



With an odd number there's an odd one out.

Brain Stretch!

Ask a question.

Get an answer.

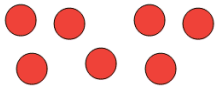
Then ask 'Why?'

Explore with Counters!

Doubling:
Choose a number to double. Do you get an even number or an odd number?

Try with other numbers.

Halving
Try halving seven.
Why doesn't it work?
Which numbers half easily? Which don't? Why?



Step 1

Splitting into Equal Groups

Children need to understand from an early stage that things can be split into different numbers of equal groups. To explore this, we take a particular number of objects and see how many different ways we can divide them equally

Learning Intentions

Understand the concept of splitting a set of things into different numbers of equal groups.

Learn the words for simple fractions.

Words

2 groups – halves
5 groups – fifths
10 groups – tenths
3 groups – thirds
6 groups – sixths
4 groups – fourths or quarters
12 groups – twelfths

Think, Do, Speak, Write: Splitting up 10

Take 10 identical objects.
(eg counters, pasta shells etc)

Divide them into **2 equal groups**. These are called **halves**. How many in each half? (5)

Point and Chorus: Half of 10 is 5

Divide the objects into **five equal groups**. These are called **fifths**. How many in each fifth? (2)

Point and Chorus: A fifth of 10 is 2

Can you make **ten equal groups**? What do you think these are called? (**tenths**) How many in each tenth? (1)

Point and Chorus: A tenth of 10 is 1

Think, Do, Speak, Write: Splitting up 6 and 12

Repeat with 6 then 12 objects to explore **halves, thirds**, **sixths**, **fourths**, & **twelfths**.

Challenge: Fractions on a Clock Face

You need a clock face and some lolly sticks.

Lay a lolly stick on the clock between the middle and the 12.
Lay another lolly stick between the middle and the 6.
How many pieces is the clock split into? (2) What is each piece called? (a **half**)

Where could you put two more lolly sticks to make 4 equal pieces (called **fourths** or **quarters**)? (from the middle to the 3 and from the middle to the 9)

Extra Challenge: Remove the stick from the 6. Can you move the others to make 3 equal pieces (**thirds**)? (possible solution: keep one pointing to the 12, move the others to 4 and 8) Can you use six lolly sticks to split the clock into **sixths**? What about **twelfths**?

Step 2

Count, Speak, Write & Draw Halves

Once you understand **what fractions are** you can start **counting, drawing & writing** them. We start with halves. **Two** halves **fit together** to make one whole. If you give all your halves away then you have **no halves**.

Speaking and writing correctly is really important. The written fraction $\frac{1}{2}$ is common on signs etc, so we start with this. First speak and **memorise: one half, 1 over 2**. You can do this like a rap.

Next explore: **no halves, 0 (zero) over 2**.; **two halves, 2 over 2**. There is always a two on the bottom because we have chopped our object into two pieces.

Learning Intentions

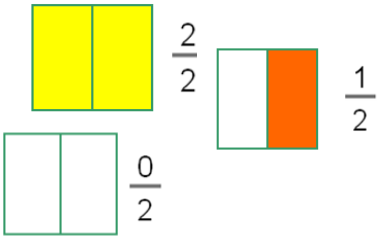
Understand 1 half, no halves and 2 halves.

Speak and write halves correctly.

Equivalences: 2 halves = 1 whole.

Draw and Write

Practise drawing and writing different numbers of halves.



Equivalences

If you have **no halves**, then you have **nothing**! You can also say **zero halves**.

For **one half** you can also say **a half**.

If you have **two halves** then you have the whole thing. We call this **one whole**.

Fun with Food

Count halves at meal times…

Rap and Clap!

1 half:

1 over 2

2 halves:

2 over 2

No halves:

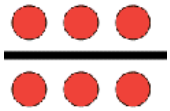
Zero over 2

Challenge with Counters

What is **one half of six?**

How many would you have if you had **no halves of six?**

What would **two halves of six** be?



Try with other numbers of counters.

Worksheet Activity

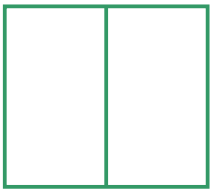
Print the worksheet on the next page. Discuss which diagrams, words and numbers match.

Play **Point and Chorus** (see page 3).

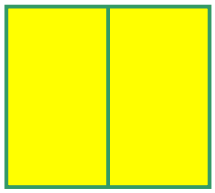
$\frac{2}{2}$

0 halves

$\frac{0}{2}$



1 half



2 halves

$\frac{1}{2}$

Step 2

Quarters and Quartering

After learning about splitting numbers into groups in Step 1, we now use quarters to explore this in more depth.

Learning Intentions

Understand what quarters are.

Understand the term 'quartering'.

Quarter groups of things.

Quartering is the same as halving and halving again.

Another word for 'quarter' is 'fourth'.

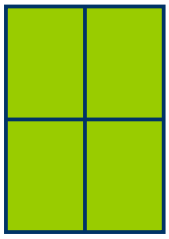
Words and Phrases

half
double
quarter
fourth

groups of
lots of
sets of

share
divide
split
each

altogether



Quartering Objects

Refer to quarters in everyday experience.

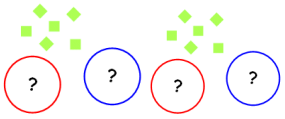
Quarter an orange.

Cut a cake into quarters.

So and Because

Use four hoops and some bean bags. Explore how to share 12 objects between four hoops.

Discuss four groups of 3 is 12 – **so** a quarter of 12 is 3, etc.



A quarter of 12 is 3 **because** 4 groups of 3 is 12.

Repeat with other numbers.

Each and Altogether

Put out four hoops.

Divide 20 bean bags equally between the four hoops.

Take turns to ask and answer the following questions.

How many **altogether?** (20)

How many **hoops?** (4)

How many **in each hoop?** (5)

Repeat with different numbers of bean bags. (eg 8, 12, 16, 24)

Investigate

Which numbers can be quartered exactly? Which ones cannot? Why?

Make a list of the numbers that will quarter. Can you see any patterns?

4, 8, 12, 16, 20, 24

Step 2

Count, Speak, Write & Draw Quarters

Writing quarters is like writing halves, except that you have a **4 on the bottom** because we have chopped our things into **four pieces**.

Learning Intentions

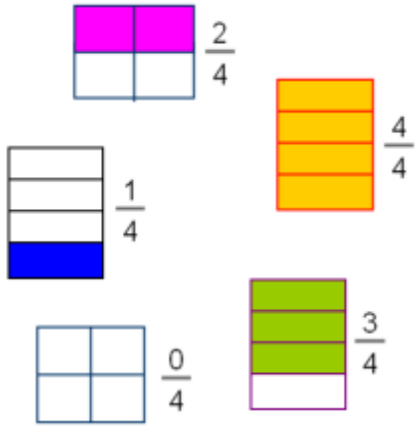
Understand 1 quarter, no quarters, 2 quarters, 3 quarters, 4 quarters.

Speak and write quarters correctly.

Equivalences: 4 quarters = 2 halves = 1 whole
2 quarters = 1 half

Draw and Write

Practise drawing and writing different numbers of quarters.



Rap and Clap!

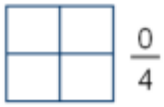
1 quarter:

1 over 4

3 quarters:

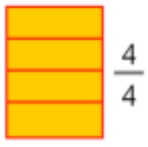
3 over 4 etc

Equivalences



If you have **no quarters**, then you have **nothing**! You can also say **zero quarters**.

Two quarters is the same as one half.



If you have **four quarters** then you have the whole thing. This is also the same as two halves.

IMPORTANT!

What Kind and How Many?

Recognise halves and quarters when they are mixed together. The number on the bottom tells us **what kind** of fraction it is (two for halves, four for quarters). The number on the top tells us **how many**.

Worksheet Activity – Point and Chorus

Print the worksheets on the next few pages. Discuss which diagrams, words and numbers match.

Play **Point and Chorus:**

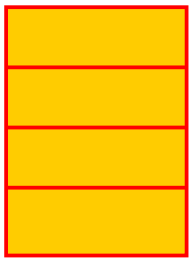
Point to each fraction and ask two questions:

First - '**What kind**'? (One word answer: 'halves' or 'quarters')

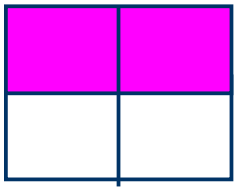
Next - '**How many**' halves/quarters? (Two word answer: zero halves, one quarter, two halves etc.)

$\frac{3}{4}$

1 quarter



0 quarters

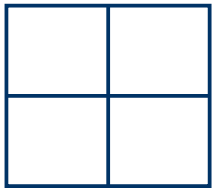


$\frac{1}{4}$

2 quarters

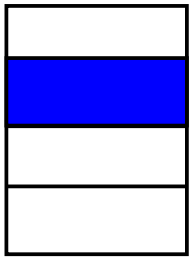
4 quarters

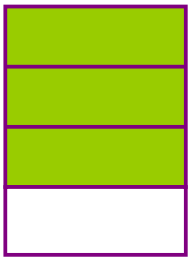
$\frac{4}{4}$



$\frac{0}{4}$

$\frac{2}{4}$

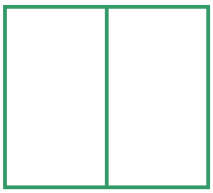




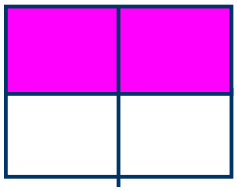
3 quarters

1 quarter

$\frac{3}{4}$

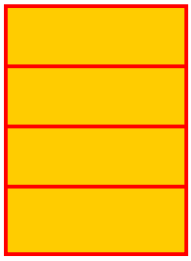


0 quarters



$\frac{0}{2}$

$\frac{1}{4}$



2 quarters

4 quarters

$\frac{4}{4}$

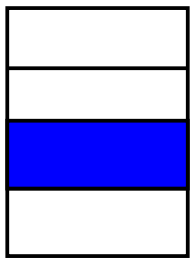
$\frac{0}{4}$

0 halves

2 halves

$\frac{1}{2}$

1 half



$\frac{2}{4}$

3 quarters

Step 2

Count, Speak and Write other Fractions

We now use shapes split into different numbers of equal pieces to explore how to speak and write fractions with different numerators. To secure the vocabulary, we look here only at fractions of shapes. Finding more complicated fractions of *groups* of things comes a little later.

Learning Intentions

Understand common fractions of shapes - thirds, fifths, sixths, eighths, tenths.

Speak fractions correctly.

Revise: What Kind and How Many?

The **bottom number** in a fraction tells us **what kind** of fraction it is (three for thirds, five for fifths).

The **top number** in a fraction tells us '**how many**' we have.

Think, Talk, Draw, Speak

Explore the different fractions you get if you colour in different numbers of parts in each diagram: eg three fifths, two thirds.

Repeat with different numbers of parts coloured in on the different sheets.

Explore and Discuss

Print several copies of the worksheet on the next page. Discuss which diagrams, words and numbers match.

Revise the fraction words

Halves, **thirds**, **quarters**, **fifths** are the tricky names

Others fit with the words – **eighths** (8 bits), **sixths** (6 bits), etc.

Quarters are sometimes called **fourths**. Halves should really be twoths(!), thirds should be threeths, fifths should be fiveths.

Rap and Clap!

1 fifth: 1 over 5
3 eighths: 3 over 8
etc

Worksheet Point and Chorus

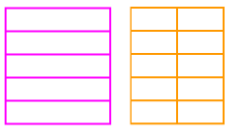
Play **Point and Chorus:** Point to each fraction on one of the sheets and ask two questions:

First - '**What kind**'? (One word answer: 'fifths' or 'thirds')

Next - '**How many**' fifths/thirds? (Two word answer: four fifths, one third, etc.)

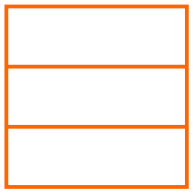
Investigate Equivalences

Shade in two fifths. How many tenths would you need to shade so that the same amount of the shape was shaded? (Answer: four tenths)



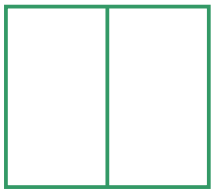
Two fifths and four tenths are called equivalent fractions. Can you find any other examples?

tenths

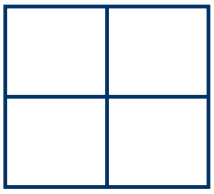


$\frac{?}{8}$

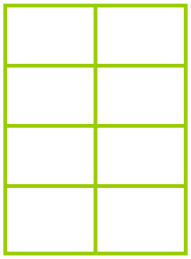
eighths



$\frac{?}{5}$



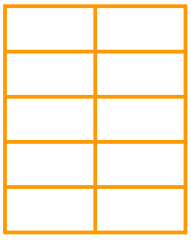
quarters



$\frac{?}{3}$

thirds

$\frac{?}{6}$

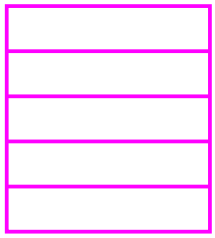


sixths

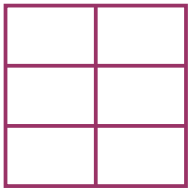
halves

fifths

$\frac{?}{2}$



$\frac{?}{4}$



$\frac{?}{10}$

fourths

Step 2

Counting Caterpillar and Tap Say Turn

To be good at fractions you need to **know your multiplication tables really well**. This requires regular practice. This page gives a brief overview of two activities which will really help with this. Once you have learned them, keep using them.

Full instructions for these games are on the Maths Investigations web site.

Learning Intentions

Count forwards and backwards in multiples of different numbers.

Memorise multiplication facts.

Multiply and Divide within the tables.

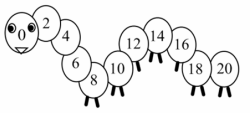
Rules for Both Games

Rule 1: If it is too easy make it a little harder.

Rule 2: If it is too hard (if you even hesitate!), make it easier.

Rule 3: If you start getting worse then your brain is tired. Time to stop!

Counting Caterpillar



Count along at a steady pace.
Cover one number at a time and repeat.
If you hesitate or make a mistake take the counter off again.
Keep going till all the numbers are covered.
Start again, counting backwards!

Tap Say Turn

2 × 2	2 × 3	2 × 4
4	6	8

Start with **one** card. Tap it. Say what is on the back. Turn it over. Tap it again. Say what is on the front. Turn it back.
Add a card at a time and repeat. **NB All cards must be turned over before turning any back.**
If you hesitate or make a mistake take the card away again.
Keep going till all the cards are on the table.

Tablet	MI Activity	Step	Related Owls
	10x table	2	Mully, Honey
	2x table	2	Divvy
	5 x table	2	Fizzy

Step 3

Halves of Metres and Mixed Numbers

The metre stick is a brilliant tool for understanding how fractions work. Ideally you want **two** of them for these activities.

Learning Intentions

There are 100cm in a metre, 200cm in 2 metres etc. 1m and 50cm is 150cm altogether.

Speak correctly – half a **metre**, one half of a **metre**, no **metres**, one **and** a half **metres**, 2 metres 30 cm.

Equivalences: half metres and cm.

Understand **mixed number** halves.

Count halves beyond one whole.

Explore and Discuss

The whole stick is 1 **metre**. (1m) The little marks show **centimetres**. (cm) There are 100 centimetres in a metre. How many in 2 metres? (200cm) How many in 0 metres? (0cm)

How long is half a metre? (50cm) What about 0 halves? (0cm) What about 2 halves? (100cm)

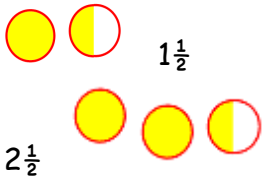
Can you get 3 halves? This will take some discussion. You have to use another metre stick! (150cm) What about 4 or 5 halves? (200cm, 250cm)

New Idea: $1\frac{1}{2}$m / $2\frac{1}{2}$m are called **mixed numbers**: made of wholes and halves mixed. You say (eg) **one and a half metres**.

Equivalences	Count Aloud
0m = 0cm	No metres
$\frac{1}{2}$m = 50cm	Half a metre
1m = 100cm	One metre
$1\frac{1}{2}$ m = 150cm	1 and a half metres
2m = 200cm	2 metres
$2\frac{1}{2}$ m = 250cm etc	2 and a half metres etc

Draw and Write

Muddle mixed numbers and their fractions and play matching games.



Tablet	MI Activity	Step	Related Owl
	Metres and Centimetres	3	Sticky
	Metres and Centimetres with Simple Halves	3	Sticky, Fizzy
	Metres and Centimetres with Mixed Number Halves.	3	Fizzy

Step 3

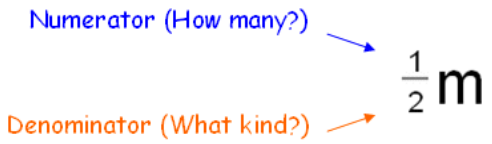
Numerator and Denominator

Before meeting improper fractions in the next activity, you may find it useful to introduce the words numerator and denominator.

Learning Intentions

'Numerator': enumerate means to count – the numerator is our counting number – it tells us '**how many**'.

'**Denominator**': means '**what kind**' of fraction it is.



Explain and Chorus

Important: *Many children confuse numerator and denominator.*

*Solution: Focus on the meaning of the words: Denominator - **WHAT KIND** of fraction; Numerator - **HOW MANY** you have. Here's how…*

First Explain: *The bottom number is the **denominator**. It tells **what kind** of fraction it is. For halves we use a 2 - two equal pieces. For quarters we use a 4 – four equal pieces.*

*The top number is the numerator. It tells us **how many** we have.*

Then Point & Chorus Activities:

Activity 1: *Point to the ceiling. Chorus: 'numerator – how many'.*

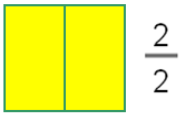
Point to the floor. Chorus: 'denominator – what kind'.

Activity 2: *Use an A3 version of the worksheet on page 18 or one of the copies of page 20 you printed and coloured.*

Leader: point to anything on the diagram and say: What kind?

Others chorus: 'Halves' or 'Quarters'.

Leader: keep pointing, say: How many?



Others chorus: '1 half', '0 halves', or '2 halves'. '1 quarter', '0 quarters', '3 quarters' etc.

Step 3

Quarters of Metres and Improper Fractions

More with the metre sticks – first exploring quarters, then improper fractions.

Learning Intentions

$\frac{1}{4}$m = 25cm

Speak correctly – quarter of a **metre**, one quarter of a **metre**, one **and** a quarter **metres** etc

Equivalences quarter metres and cm.

Understand **improper fractions**.

Count halves and quarters beyond one whole, in different ways.

Explore and Discuss with Blu Tac

Where would you have to chop to cut the metre stick into **four equal pieces?**

Where would $\frac{1}{4}$ of a metre be? (25cm)
What about 0, 2, 3, 4 quarters of a m?

Can you get 5 quarters? Yes! (This would be $1\frac{1}{4}$ metres = 125cm) 6 quarters? ($1\frac{1}{2}$m = 150cm) 7 quarters? ($1\frac{3}{4}$m = 175cm)

For five quarters you can write $\frac{5}{4}$
This is called an **improper fraction**. ($\frac{1}{4}$, $\frac{3}{4}$ etc are called **proper** fractions.)

Equivalences

0m = 0cm
$\frac{1}{4}$ m = 25cm
$\frac{1}{2}$ m = 50cm
$\frac{3}{4}$ m = 75cm
1m = 100cm
$1\frac{1}{4}$ m = 125cm
etc

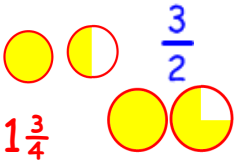
Count Aloud

No halves, 1 half, 2 halves, 3 halves etc.

Zero, a quarter, a half, 3 quarters, one whole, one and a quarter, one and a half, 1 and $\frac{3}{4}$ etc.

Point and Chorus

Mixed number and improper fraction halves and quarters and their diagrams.



Tablet	MI Activity	Step	Related Owl
	Metres & Centimetres with Quarters.	3	
	Metres & Centimetres with Halves and Quarters.	3	Quarty
	Metres and Centimetres with Two Kinds of Halves.	3	
	Metres and Centimetres with Many Quarters.	3	Quarty

Step 3

Quadruple and Quarter Numbers

Now that we know how quarters work, we switch back to using counters and further explore quartering (and quadrupling) of **numbers** and how this relates to **dividing and multiplying.**

The word 'quadruple' is not one we would normally use with young children, but it is here for completeness. If you prefer, you can talk about '4 lots' or '4 groups'.

Learning Intentions

Understand the terms "quartering' and 'quadrupling'.

Quartering is halving and halving.
Quadrupling is doubling and doubling.

Halving = divide by 2.
Double = multiply by 2.
Quarter = divide by 4.
Quadruple = multiply

Revision

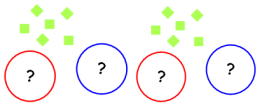
Recap the activities from Step 2 Quarters and Quartering.

Symbols

x multiply (make more)

÷ divide (split up)

Explore, Discuss, Write



Explore, with counters, different ways of describing eg. 4 **lots of** 3 is 12, 4 **times** 3 is 12, 4 **multiplied by** 3 is 12, 4 **x** 3 = 12, **quarter of** 12 is 3, $\frac{1}{4}$ **of** 12 is 3, 12 **divided by** 4 is 3, 12 ÷ 4 = 3.

Lots of practice speaking and writing! Repeat with other numbers.

Each and Altogether

Divide 24 counters equally. Ask and answer the following questions: How many **altogether?** (eg 20) How many **groups?** (eg 4) How many **in each group?** (eg 6). Record in writing. 20 ÷ 5 = 4, $\frac{1}{4}$ of 20 = 5, 5 x 4 = 20.

Repeat with different numbers of counters. (eg 8, 12, 16, 20, 28)

Play Counting Caterpillar and Tap Say Turn for the 4x table.

Tablet	MI Activity	Step	Related Owl
	Quartering Numbers in the 4x Table	3	Quarty
	4x table	3	Quarty
	Divide within the 4x table	4	Quarty

Factors

The **factors** of a number are the numbers that you can multiply together to make it. They play a really important role in fractions. These activities introduce them. We will see how they relate to fractions later.

Learning Intentions

Be able to explain what factors are using rectangular arrays.

So and Because

The factors of a number are the numbers that multiply to make it.

2 x 5 = 10 *so*
2 & 5 are factors of 10.

2 & 5 are factors of 10.
Because 2 x 5 = 10

Think,Talk and Do: Factors of 10

Arrange ten counters in two rows like this.

How many rows? (2) How many in each row? (5)
Point and Chorus: '2 rows of 5 is 10'. 2 x 5 = 10.

Turn the counters this way.
How many rows? (5) How many in each row? (2)
Point and Chorus: '5 rows of 2 is 10'. 5 x 2 = 10.

Put out another set of 10 counters like this.

Point and chorus again: 1 row of 10 is 10. 1 x 10 = 10.
Turn the other way. 10 rows of 1 is 10. 10 x 1 = 10.

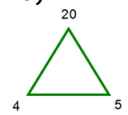
New words: 1, 2, 5, 10 are called the **factors** of 20.
You can arrange them in a **factor rainbow**.

Think, Talk, Write: Factors of 20

Repeat everything with 20 counters.
Write down all the multiplication facts.
1 x 20 = 20, 20 x 1 = 20
2 x 10 = 20, 10 x 2 = 20
4 x 5 = 20, 5 x 4 = 20

Investigation: Factor Triangles

Factor pairs (4 x 5 = 20, 5 x 4 = 20) can be shown in a triangle:



How many factor triangles can you make for the number 20?

Explore Further: Factors & Primes

Use counters to investigate factor triangles and rainbows for other numbers. Which numbers have several factors? (12, 15, 18 etc) Which have only two? (7, 11 etc) Numbers like 7, where the counters can only be arranged in a single line are called **prime** numbers. Prime numbers have only 2 factors. What other prime numbers can you find? [Interesting fact: The number '1' is not a prime number because it has only 1 factor, not 2!]

Tablet	MI Activity	Step	Related Owl
	Factors of 10 and 20.	3	Mully

Step 4

Single Place Decimals: Tenths and Halves

Decimals are very closely linked to fractions. To understand fractions well, you need to understand their equivalences with decimals. The **metre stick** is really good for introducing the idea of decimals. We first explore **tenths**.

Learning Intentions

The coloured sections of the metre stick are called 'tenths' of a metre *because there are ten of them.*

0.1m means 0 whole metres and 1 tenth.

0.2m means 0 whole metres and 2 tenths, etc.

0.5m (5 tenths) is the same as ½m (half a m)

Equivalences

Where is ½m? (at 50cm)
How many fifths? (5 fifths)
What would ½ a metre be as a decimal? (**0.5m**)

Think & Talk

Where would you cut to chop the stick into **two equal pieces?** (at 50cm). What fractions are these? (**halves**).

Where would you cut to chop the stick into **ten equal pieces?** (at 10, 20, 30cm etc – where the colours change) What fractions does this make? (**tenths**). The tenths are the coloured 'bits' on the stick.

To get to 40cm, how many 'bits' would it be? (4 bits)
To get to 20cm? (2 bits – 2 tenths)

Think, Talk, Write

We can write one tenth like this. 0.1
We say **zero point one**. This is called a **decimal**.
The **zero** shows we have **no whole metres** and the **one** shows we have **one 'bit' or tenth**.

How would you write two tenths as a decimal? (**0.2**)
How would you say it? (**zero point two**) etc.

Tablet	MI Activity	Step	Related Owl
	Metres and Centimetres with Halves and Tenths	4	Dessy
	How Decimals Work - Tenths	4	Dessy
	Decimal Halves	4	Dessy
	Decimal Tenths and Halves	4	Dessy

Decimals & Equivalences: Tenths and Fifths

The question '**Where would you need to cut to chop the stick into five equal pieces?**' leads naturally on to exploring **fifths** and their decimal equivalents.

Learning Intentions

1.1 means 1 whole and one tenth.
$1\frac{1}{2}$ m = 1.5m

0.2 (2 tenths) is the same as 1 fifth.

Fifths are twice the size of tenths.

Equivalences

How long is one fifth of a metre? (20cm)
How many tenths is that (2 tenths)
We say **1 fifth is equivalent to 2 tenths**.

What other equivalences can you find?
(2 fifths = 4 tenths
3 fifths = 6 tenths etc)

Think & Talk

How long is 9 tenths of a metre? (90cm)
How long is 10 tenths (1 whole metre)
Can you get 11 tenths? (Yes – you need another stick)
1 whole metre & 1 tenth - we write this as **1.1**.

What would 1.2 mean? (1 whole metre and 2 tenths = 12 tenths) etc.

Fifths with Blu Tac

Where would you cut to chop the metre stick into **ten equal pieces?** (10cm, 20cm, 30cm etc). What fractions are these? (**tenths**).

Where would you cut to chop the stick into **five equal pieces?** (20cm, 40cm, 60cm etc) What fractions are these? (**fifths**).

So **1 fifth of a metre = 20cm = 0.2m**.

What would 2 fifths of a metre be? (40cm = 0.4m)
What about 3 fifths, 4 fifths or 5 fifths?

Think & Talk Which is bigger, a fifth or a tenth? Why?

Tablet	MI Activity	Step	Related Owl
	Metres and Centimetres with Many Tenths	4	
	Metres and Centimetres with Tenths and Fifths	4	Dessy
	Decimal Tenths and Fifths.	4	Dessy
	Counting in Decimal Tenths, Halves and Fifths	4	Dessy

Step 4

Fractions of 12 and 20

We now get out the counters again and explore finding **a range of fractions of small numbers**. This work links really nicely to the work your children will also have been doing at this level on **division**, using **rectangular arrays**.

Learning Intentions

Find halves, thirds, quarters, fifths, sixths, tenths of small numbers.

Revision: Step 2 'Thirds, Fifths, Sixths, Tenths, Eighths' & Step 3 'Quarter and Quadruple Numbers'

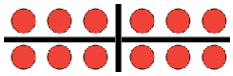
Equivalences

What is $\frac{1}{2}$ of 12? How does this compare with 2 quarters? (It's the same) What about 0 halves / 0 quarters? What about 2 halves / 4 quarters?

What equivalences can you find for thirds and sixths?

Halves & Quarters

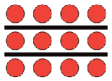
Spend time on this: Explore/discuss carefully



Split 12 counters into 4 groups.

How many in **each quarter** of 12? (3) So what would **two quarters of** 12 be? (6) **$\frac{3}{4}$ of** 12? (9) **4 quarters of** 12? (12) **0 quarters of** 12? (0)

Thirds & Sixths



Split 12 into **three** groups, then **six**.

Explore thirds & sixths of 12.

Rap and Clap

1 quarter of 12 is 3.
2 quarters of 12 is 6.
3 quarters of 12 is 9.
4 quarters of 12 is 12.

Repeat with thirds, sixths etc.

Fifths & Tenths

Repeat everything with 20 counters. Explore halves, quarters, fifths, tenths.

Explore Further

Use counters to investigate fractions of other numbers. Which numbers are best for dividing up in different ways? (15, 24, 27, 30, 36) Which don't work at all? Why? (Numbers like 19 are not in any 'times table' so they don't divide up nicely)

Tablet	MI Activity	Step	Related Owl
	Fractions of 12	4	Fracto
	Fractions of 20	4	Fracto

Keep playing Counting Caterpillar & Tap Say Turn: 2, 10, 5, 4, 3x tables.

Step 4

Unit Fractions Using Tables

Now that we have thoroughly explored the **ideas** of fractions using counters, we can start finding mental shortcuts using our knowledge of multiplication tables. We begin with finding **unit fractions** – fractions with a numerator of 1.

Learning Intention

Find **unit fractions** of numbers using the multiplication tables.

Understand the links between multiplication, division and finding fractions of things.

So and Because

20 ÷ 5 is 4 **so** 1 fifth of 20 is 4.

1 tenth of 20 is 2. **because** 20 ÷ 10 is 2.

etc

Think and Talk

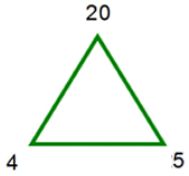
Arrange 20 counters like this.

What different calculations can you write for this diagram?

4 rows of 5 is 20 4 x 5 = 20.

20 ÷ 4 = 5 $\frac{1}{4}$ of 20 = 5

5 out of 20 = $\frac{1}{4}$



Turn the counters the other way.

5 rows of 4 is 20 5 x 4 = 20

20 ÷ 5 = 4 1 fifth of 20 = 4

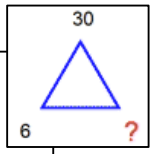
4 out of 20 = 1 fifth

Challenge:
Can you use tables facts to find unit fractions of other numbers without counters?

1 third of 24 = 8 **because** 3 x 8 = 24.

1 sixth of 30 = 5 **because** 5 x 6 = 30.

Investigation



How many different factor triangles can you find for 30? What are the fraction facts that go with them?

Tablet	MI Activity	Step	Related Owl
	Find Unit Fractions Using Multiplication Facts	4	Fracto
	Divide within the 4x table	4	Fracto
	Divide within the 3x table	4	Fracto

Keep playing Counting Caterpillar & Tap Say Turn!

Step 4

Fifths and Tenths Using Tables

If you can find unit fractions successfully, then it is a small step to finding any fraction of a number. To keep the numbers easy while we learn the process, we work at this level with fifths and tenths. Note that this is the first time we are *formally learning a two-step process*. Make sure that the underlying concepts are really secure before you do this.

Learning Intention

Use dividing and then multiplying to find **various fifths and tenths** of numbers in the appropriate tables.

Tenths of 20

Repeat the activity on the right with the counters arranged in pairs like this so that you can find tenths.

Think and Talk – Fifths of 20

Arrange 20 counters like this again.

What is one fifth of 20? (4)

Why? (Because 5 groups of 4 make 20, 5 x 4 = 20, 20 ÷ 5 = 4)

What is 3 fifths of 20? (12)

Why? (Because one fifth of 20 is 4 so three fifths will be 3x bigger, 3 x 4 =12)

So to find 3 fifths of 20 we first divide by 5 to find one fifth and then multiply by 3 to find 3 fifths.

20 ÷ 5 = 4 3 x 4 =12

Repeat the discussion to find 4 fifths of 20, 2 fifths of 20 in the same way.

Challenge:
Use tables facts to find fifths and tenths of other numbers without counters.

3 fifths of 40 = 24 **because** 1 fifth of 40 = 8 and 3 eights are 24. (40 ÷ 5 = 8, 3 x 8 = 24)
7 tenths of 50 = 35 **because** 1 tenth of 50 = 5 and 7 fives are 35. (50 ÷ 10 = 5, 7 x 5 = 35)

Investigation - Equivalences

What are 3 fifths of 30? (1 fifth is 6 so 3 fifths is 18)
What are 6 tenths of 30? (1 tenth is 3 so 6 tenths is 18)
What do you notice? (They are the same)
Why does this work? (3 fifths is the same as 6 tenths)
How many other paired facts can you find like this?

Tablet	MI Activity	Step	Related Owl
	Find Fifths & Tenths using Tables	4	Fracto

Keep playing Counting Caterpillar & Tap Say Turn!

Step 4

Something out of Something as a Fraction

As well as being able to find **fractions of** numbers, you need to be able to work backwards to express the relationship between two numbers **as a fraction**. We explore this next.

Learning Intentions

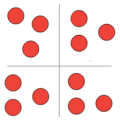
Express (eg) 5 out of 20 as a fraction. ($\frac{1}{4}$)

Understand the link between 'fractions of' and 'something out of something' as inverses.

New Idea - Inverses

The **inverse** of something means its **opposite**.
The inverse of closing a door is opening it!
The inverse of tying your shoelaces? Untying them!
What is the inverse of adding? (subtracting)
What is the inverse of multiplying? (dividing)
'Find something out of something **as a fraction**' is the inverse of 'find a **fraction of** something'.

Think and Talk



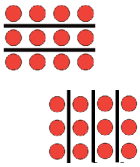
Split 12 counters into 4 groups.

Circle three out of the 12: how many quarters of 12 is that? ($\frac{1}{4}$ - because $\frac{1}{4}$ of 12 = 3)

Consider 9 out of the 12: what fraction of 12 is that? ($\frac{3}{4}$ - because $\frac{3}{4}$ of 12 = 9)

What about 12 out of 12? (4 quarters, 2 halves) Or 0 out of the 12? (0 quarters, 0 halves)

Challenge: Two diagrams side by side: thirds & quarters. What is 8 out of 12? (2 thirds) What is 9 out of 12? (3 quarters) etc. Extend to include sixths!



Inverse Rap and Clap – Because

4 out of 12 is 1 third **because** 1 third of 12 is 4.
8 out of 12 is 2 thirds **because** 2 thirds of 12 is 8.
12 out of 12 is 3 thirds **because** 3 thirds of 12 is 12.

Inverse Point & Chorus – So

$\frac{1}{4}$ of 12 is 3 **so** 3 out of 12 is $\frac{1}{4}$.
$\frac{1}{2}$ of 12 is 6 **so** 6 out of 12 is $\frac{1}{2}$.
$\frac{3}{4}$ of 12 is 9 **so** 9 out of 12 is $\frac{3}{4}$.

Explore More

Repeat with 20 counters: halves, quarters, fifths, tenths. Then try other numbers.

Tablet	MI Activity	Step	Related Owl
	Something out of twelve as a fraction;	4	Fracto
	Something out of 10 or 20 as a fraction	4	Fracto

Keep Playing Counting Caterpillar and Tap Say Turn!

Step 4

Halves and Quarters with Remainders

We have already touched on what happens if things do not divide exactly. We now explore this properly and look at different ways of recording the answer.

Learning Intentions

Explain what happens when you try to half an odd number or quarter a number which is not a multiple of four.

Express answers to divisions as remainders or fractions.

Think, Talk, Write

Put out 7 counters. What happens if you try to split them into two equal groups? What is **half of 7?**

There are two ways to do this: (1) Suppose the counters are chocolate bars. Split them into two groups of three and imagine the other one cut with half in each group.
$\frac{1}{2}$ of 7 = $3\frac{1}{2}$ (or as a decimal, 3.5)

(2) Suppose the counters are dogs. You can't cut dogs in half! Split into two equal groups with one left over. We call this a **remainder**. For 'remainder' write **r**.
$\frac{1}{2}$ of 7 = 3 r 1

Repeat with 9 counters.

Challenges: Quarters

1. What happens if you try to find $\frac{1}{4}$ of 9 counters?

(Either 2 in each group and one left over: $\frac{1}{4}$ of 9 = 2r1 or $2\frac{1}{4}$ in each group: $\frac{1}{4}$ of 9 = $2\frac{1}{4}$. Avoid decimals here – we haven't covered two-place decimals yet!)

2. What happens if you try to find $\frac{1}{4}$ of 10 counters?

(Either 2 in each group and two left over: $\frac{1}{4}$ of 10 = 2r2 or split each of the spare ones in half: $\frac{1}{4}$ of 9 = $2\frac{1}{2}$.)

3. What happens if you try to find $\frac{1}{4}$ of 11 counters?

(Either 2 in each group and three left over: $\frac{1}{4}$ of 11 = 2r3 or put $\frac{1}{4}$ of each of the spare ones into each group so you get three quarters in each: $\frac{1}{4}$ of 11 = $2\frac{3}{4}$.)

Investigation

What other numbers give a remainder when you half them? (all odd numbers) Why? (because they have an odd one out)

Which numbers give you a remainder of 1 when you quarter them? Which give a remainder of 2 (or 3, or 4)? Is there a pattern?

Tablet	MI Activity	Step	Related Owl
	Finding Half of Odd Numbers	3	Fracto
	Finding Quarters with Remainders	4	Fracto

Appendices

That's all the steps for now. Here are the extra bits promised at the beginning to help you make the most of everything…

Techniques for Learning

Practical activity is really important for learning. The following simple techniques can be used at any stage. Some are referenced directly in the step by step guide. All can be used whenever you wish. An overview is given here - some are explained more fully in the individual pages.

For maximum effect, use regularly, repeatedly, in combination with each other.

For use at home or in school:

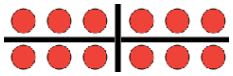
Practical equipment: Use counters, metre sticks, or diagrams/calculation patterns on individual whiteboards, to explain concepts.

Thinking questions: Use questions to develop thinking and steadily build understanding of the concept you are teaching.

- **The 'Why' and 'How' questions:** If a child answers a question, ask 'Why?' or 'How?' to get them to justify the answer. Keep asking 'Why?' of 'How?' to get deeper thinking.
- **What would happen if?** Get children to build on what they know to explore new ideas. *Examples:* You know what 1 half and 2 halves are: can you get 3 halves? What would happen if you tried to find a quarter of 11?

Point and Chorus: Using practical equipment, point and model the appropriate phrase(s). Child(ren) repeat(s), pointing to reinforce. Repeat over and over.

Example: 12 counters split into 4 groups.



Draw with a finger around one group of three counters and say $\frac{1}{4}$ of 12 is 3. Draw round two groups together and say 2 quarters of 12 is 6. Draw round three groups and say $\frac{3}{4}$ of 12 is 9, etc.

So and because: Build phrases using 'so' or 'because'. Make a big play of the word 'so' or the 'because'. Child(ren) repeat(s). *Examples:* 4 lots of 3 is 12 *sooo* $\frac{1}{4}$ of 12 is 3; $\frac{1}{2}$ of 14 = 7 *becaauuse* 14 ÷ 2 = 7 etc.

Rap & Clap: Clap or snap fingers to create a rhythm and chant a pattern you have discovered. *Example:* $\frac{1}{2}$ of 12 = 6, $\frac{1}{2}$ of 14 = 7, etc.

Verbal ping-pong: Give a rule for generating an answer to a single number question. Establish a rhythm - you and child(ren) alternate. *Example:* equivalences $\frac{1}{2}$ m : 100cm, $\frac{3}{4}$ m : 75cm, 0m : 0cm, etc,

For groups of children in school

Gather: Gather the children round a table for your teaching input, as you might do for an art lesson. With a bit of judicious arranging – some sitting, some kneeling, some standing, you can make sure that everyone can see clearly what you are doing.

Choose the next person: Each pupil who answers chooses the next person to answer.

Phone a Friend: Allow pupils to 'phone a friend' if stuck answering a question. The friend then answers instead.

Practical Equipment

Practical experience of number is really important in the early stages. To use the activities successfully, practical equipment is essential. It is all easily available or easy to make.

'Counters' Big and Small

Hoops and bean bags are suggested in one activity. If you don't have these, use big sheets of paper and toy cars or teddies. For 'counters' you can also be creative. At home, the kitchen will provide lots of possible objects to count. Coloured pasta shells are particularly good!

Metre sticks

For the more advanced activities it will really help to have **metre sticks with just the tens and fives numbers marked**.

At home, it would be good to have **two sticks**. If you wish, you can make your own. Your hardware store should be able to cut a strip of wood into 1m lengths. First, use a tape measure to mark where the 10cm divisions are and write the 10s numbers on. (10, 20, 30 etc) Then mark divisions (but not the numbers) for all the in-between numbers. (1, 2, 3, etc) Finally, write the 5s numbers too, at the half-way points. (5, 15, 25 etc)

In school, you ideally want **one stick between two**. Buy several class sets and then share around when needed. These sticks are also brilliant for teaching addition and subtraction, so they will be well used!

You can order ready-made ones from the Ruler Company. These are particularly good because the alternate tens are coloured – this helps in discussion. (http://www.rulerco.co.uk/index.php?doc=13&vid=60).

Individual Whiteboards & Pens

A set of wipeable A4 sized boards is really useful for writing things during discussion sessions.

Thinking and Talking

It is really important for children's understanding is to **get them to explain their thinking** (using practical equipment to demonstrate).

'Why?' 'How?' and 'What would happen if?'

'Why?' and **'How?'** are the most important questions in the world, because they challenge you to think! The more you ask them of your children, and the more they ask them of themselves, the more they will learn.

When you get an answer, ask 'Why?' **How** did you work that out? **Why** does it work like that? You can also use '**What would happen if?**' to stimulate further thinking.

So and Because

The words 'so' and 'because' are particularly useful. You may enjoy playing around with these and make a big thing out of using them:

6 x 2 = 12
sooo $\frac{1}{2}$ of 12 = 6.
sooo $\frac{1}{2}$ of 120 = 60, etc.

$\frac{1}{4}$ of 24 = 6
becaauuse 24 ÷ 4 = 6
becaauuse 4 lots of 6 is 24, etc.

Ideas before Rules

It is really important when helping children with Maths to resist the temptation to say: 'This is how you do it. Now do some more to practise.'

For real learning to take place, **understanding** must come first. This is where the **practical equipment** is so important, because it helps you to **see** number facts and relationships and not just learn them by rote. In the step-by-step guide that follows, for example, the mental **process** for finding fractions of numbers is not learned until late in step 4, Lots of practical experience comes first!

Learning is not a race. Don't rush!

Tips for Helping with Maths at Home

Successful learning is all about self confidence.

If you are working with your child at home, remember these four golden rules. Your child's confidence will grow steadily and you will enjoy your learning sessions together.

Four Golden Rules

Be Positive
Praise continuously - lots of it! Let your child feel your delight in his or her progress.

Revise frequently
It is important to revise regularly the key skills your child has learned so far. Regular revision in frequent short bursts is the best.

Take things at a steady pace
Learning takes place in small steps, each following on from the one before. Successful teachers encourage their pupils to develop one step at a time. Waiting until a child has thoroughly mastered the present step before leading on to the next produces a child who is enthusiastic and confident about learning.

Take regular breaks
There is a limit to the length of time anyone can concentrate! Every child's concentration period is different. Use your own judgement as to when to stop. If you feel frustrated with your child or your child begins to do worse you have gone on too long!

The touchy feely bit...

Certain personal qualities (both for you and your child) will come in very useful if you are doing Maths together.

A SENSE of HUMOUR: (Both of you)
Remember that some learning can be a challenge. It is important to keep cheerful and not fall out with people!

COURAGE, and a WILLINGNESS TO TAKE RISKS. (Your child)
You have to be prepared to TRY and to be WILLING TO MAKE MISTAKES. It is through our mistakes that we often learn most.

KINDNESS and PATIENCE. (You!)
We all learn at different speeds and we learn best when we can ask questions, and puzzle over things we don't understand, without feeling that others are going to get annoyed with us.

It is also helpful for us as parents to remember that we have TWO EARS for listening and only ONE MOUTH for speaking. Remember that your child will learn most if (s)he discusses what (s)he is thinking, explores ideas and discovers things for him/herself, rather than you trying to explain how to do everything.

Understanding Fractions

There are two parallel strands of learning that need to happen in the early stages if children are to understand how fractions work.

1. Children need to understand **what fractions are**, **how they are written**, and the fact that the same fraction can be written in different ways. This leads on naturally to an exploration of **where fractions lie on the number line**, how fractions compare, and an investigation of their **equivalences** with each other and with other things: whole numbers, decimals and (later) percentages.

2. Children need to understand the concept of finding a **fraction of** another number and the fact that this is closely connected with the concept of division (and therefore also multiplication). This then leads on to the inverse (reverse) process: how to **express one number as a fraction (or percentage) of another**.

At a much later point, children need to explore how to **calculate** using fractions; learning how to add, subtract multiply and divide with varied selections of whole numbers, fractions, mixed numbers and decimals.

Further Reading

Part 2 of this guide will follow, taking these concepts and ideas further.

Meanwhile, for those who would like to explore wider aspects of the topic, the following links to research into children's learning of fractions may be of interest:

1. http://dera.ioe.ac.uk/22312/1/doc_3085.pdf
2. http://journals.plos.org/plosone/article/asset?id=10.1371/journal.pone.0080016.PDF