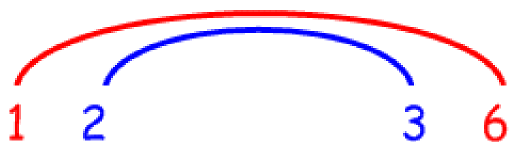
Multiplication Rectangles
& Factor Rainbows

What's this one all about?

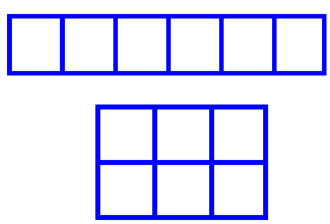
A factor rainbow is a beautiful idea that deserves to be part of every child's Mathematical vocabulary. Here's the factor rainbow for 6:



The factors of 6 are the numbers that divide into 6 exactly: 1, 2, 3 & 6. In the rainbow they come in order of size and each is linked by a bow to its partner.

You can make 6 by multiplying its factors together like this:
1 x 6 = 6   2 x 3 = 6

The multiplication rectangles for 6 look like this:



One row of 6 and two rows of 3.
1 x 6 = 6   2 x 3 = 6

You get the idea!

In this investigation we use these two ideas to investigate the factors of lots of different numbers and, in so doing, learn lots about multiplication facts and how they connect together.

The teacher bits…

Learning Intentions: I understand that multiplication can be understood diagrammatically using a rectangle. I can use factor rainbows to work out the factor pairs for different numbers. I understand that some numbers have lots of factors. I understand that square numbers have an odd number of factors. I understand that some special numbers have lots of factors and can be divided up in lots of ways.

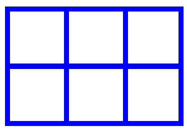
Age: 7-12

What you need: Counters, squared paper.

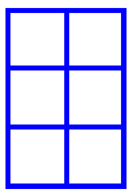
The investigation

Class Discussion – Explaining the Concepts

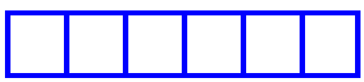
Ask the pupils how many different ways there are of making 6 by multiplying two numbers together. Build a list and show how the how the four facts can be represented using rectangles.



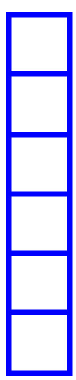
2 x 3 = 6



3 x 2 = 6

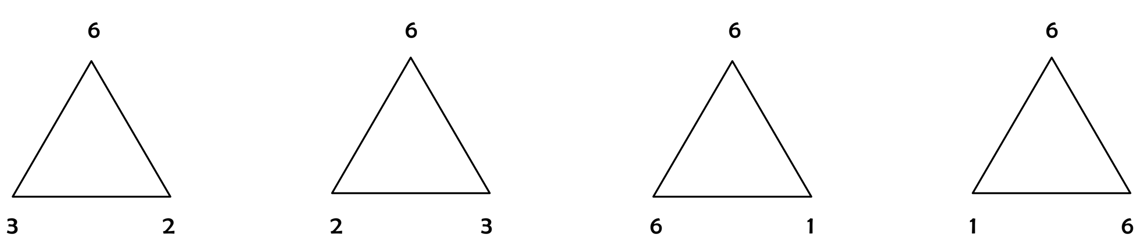


1 x 6 = 6



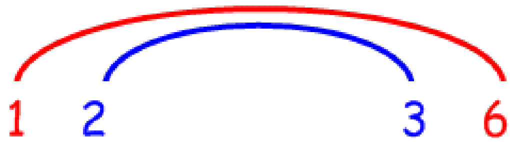
6 x 1 = 6

Show the link to multiplication triangles and note that there is one triangle corresponding to each rectangle.



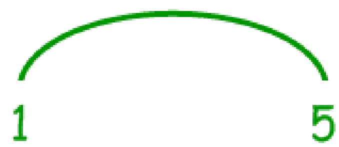
Explain that the numbers that multiply to make six (1, 2, 3, 6) are called the *factors* of 6.

Show how to draw the *factor rainbow* for 6. Factor rainbows are powerful because they simultaneously show the factors in order and show which factor multiplies by which to make the parent number.



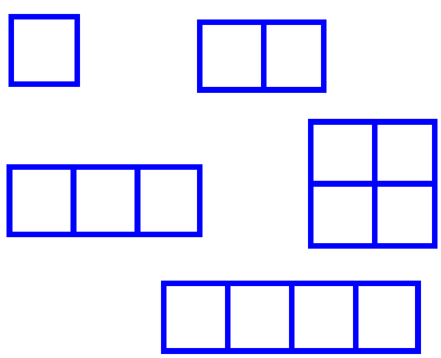
Drawing factor rainbows requires a little skill. You get the best result if you write the numbers in order first, equally spaced, and then put in the loops. Make sure you show the children how to do this or you will get some bizarre results!

Ask what the factor rainbow for 5 would look like. Agree that the only factors of five are 1 and 5. (1 x 5 = 5 and 5 x 1 = 5). So the factor rainbow is a little bit boring. It looks like this!



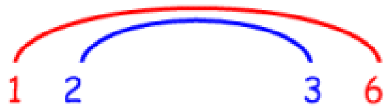
Initial Puzzle

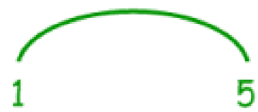
Ask the children how many different rectangles (or squares) they think they could make with 1 square. What about 2 squares, 3 squares, 4 squares? What would their factor rainbows look like?



Make sure the children are clear that composite squares (eg 2 x 2 = 4) are allowed as well as rectangles. (If appropriate, you could explore the idea that a square is actually a special kind of rectangle.)

What about larger numbers? (7, 8, 9, 10 etc) How many ways can you make them by multiplying? How many different rectangles could you make? What would their factor rainbows look like?





Pair or Team Work

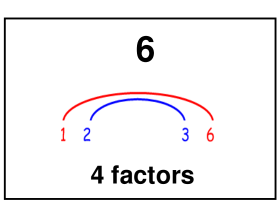
Set the pupils to work in pairs or teams to investigate factors for all the numbers up to 20. Which numbers have only two factors? (like 5) Which numbers have more than two factors? (like 6) Are there any numbers with only one factor? To record their thinking, they could draw rectangles and/or draw factor rainbows. They could also record their results in a table like this.

Number	Factors	Number	Factors
1	1	11	
2	1, 2	12	
3	1, 3	13	
4	1, 2, 4	14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

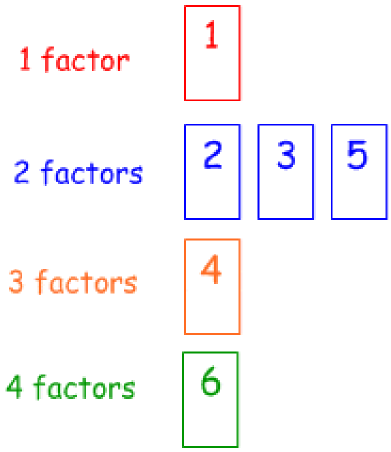
Circulate and support as they work. Any pupils doing particularly well could be challenged to investigate all the numbers up to 30. If you wish, you can have the pattern builder loaded up on the interactive board, or have a number of ipads or laptops available so that pupils can check their answers as they go.

Plenary

Gather the pupils together in a circle on the carpet, or round a table, and discuss results. Go through each number in turn, agreeing together what the factors are. Once each number has been agreed, delegate one pupil to write the number on an A4 whiteboard, draw its factor rainbow, and write the total number of factors.



Once several of the whiteboards have been completed, begin to arrange them in a giant pattern on the floor or table. Put the numbers with one factor in the first row, put the numbers with two factors in the next row, and so on. What do they notice that is interesting?



A number of interesting facts may emerge in discussion:

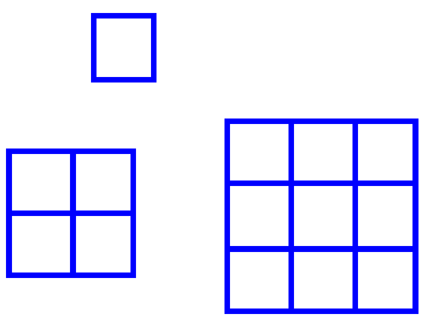
1. There is only one number with one factor: the number 1.

2. The row with the most numbers is the 'two-factors' row. (These are actually the prime numbers, and are investigated fully in another investigation.)

3. There are far more numbers in the even numbered rows than in the odd numbered rows. (ie most numbers have an even number of factors.)

Further Investigation – Square Numbers

Some numbers can be arranged in squares. These are called square numbers.



How many square numbers can the pupils find? How many factors do they have? What is interesting about their factor rainbows?

[The square numbers all have an odd number of factors. The factor rainbow for a square number can be drawn by looping the middle number onto itself.

eg 6]

Further Investigation – Numbers with Lots of Factors

Which 2-digit numbers have the biggest factor rainbows (the most factors)?

Further Investigation – Bus Timetables

Buses often come at the same number of minutes past each hour. For example, a bus that comes 4 times an hour might come at 10:00, 10:15, 10:30, 10:45, 11:00, 11:15, 11:30, 11:45 etc.

You can show this as 00, 15, 30, 45.

Make a table to investigate other frequencies of service. Look at real bus timetables and compare the numbers.

Use the pattern builder for adding to check what happens when you have different starting numbers.

eg

10 + 12 = 22

22 + 12 = 34

34 + 12 = 46

46 + 12 = 58

58 + 12 = 70 (minus 60 takes you back to 10)

So the minutes are 10, 22, 34, 46, 58.

Further Investigation – Multiplication Square

Get a multiplication square. Work out how many factors each number has.

1 has only one factor. Colour it red.

Find all the numbers with two factors. Colour them blue.

Find the numbers with three factors. Colour them orange.

Find the numbers with four factors. Colour them green.

Find the numbers with five factors. Colour them pink. And so on.

What patterns do you get? Why?

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Further Investigation – Dividing a Circle

60

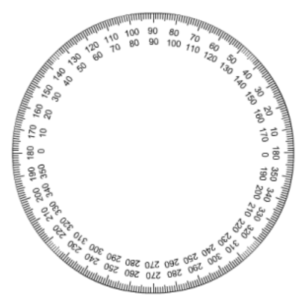
12

24

How many months are there in a year? How many hours in a day? How many minutes in an hour?

What is special about the factor rainbows for these numbers?

Why do you think people chose these numbers to measure time?



Degrees are used to divide up a circle.

Find a circular protractor. How many degrees are there in a whole circle?

What is special about the factor rainbow for this number?

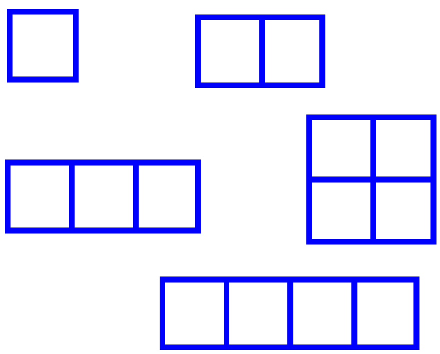
Why did ancient people choose this number to divide up circles?

[360 divides by 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360. So it is a brilliant number to use for dividing a circle, because it means that (with particular reference to the circle of the 'heavens' with all the stars etc) it can be divided up in numerous ways. It must also have not escaped the notice of ancient astronomers that number 360 is very close to 365 – the number of days in a year. Hence the number 360, will have held special significance. All the other special numbers mentioned earlier are factors of 360.]

Name: ____________________ Class: _____ Date: ____________

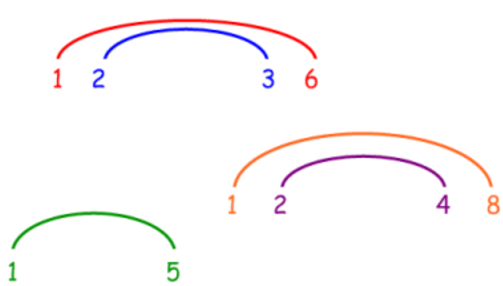
Multiplication Rectangles and Factor Rainbows

How many different rectangles (or squares) can you make with 1 square, 2 squares, 3 squares, 4 squares, 5 squares, 6 squares, more squares?



Investigate all the numbers to 20. How many ways can you make each one by multiplying? How many factors does each one have?

What would the factor rainbows look like?



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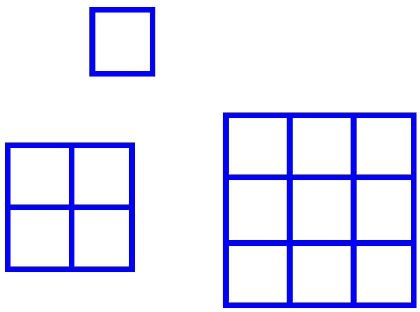
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

Record your discoveries in a table.

Name: __________________ Class: ____ Date: ___________

Square Numbers

Some numbers can be arranged in squares. These are called square numbers.



How many square numbers can you find?

How many factors do they have?

What is interesting about their factor rainbows?

Name: ________________ Class: ____ Date: __________

Number with lots of Factors

Which numbers have the biggest factor rainbows (the most factors)?

Try these to start with…

What others can you find?

Name: __________________ Class: ____ Date: ___________

Bus Timetables

Buses often come at the same number of minutes past each hour.

For example, a bus that comes 4 times an hour might come at

10:00, 10:15, 10:30, 10:45, 11:00, 11:15, 11:30, 11:45 etc.

You can show this as 00, 15, 30, 45.

Make a table to investigate other frequencies of service. Make all the buses come 'on the hour' (00) for their first time.

Frequency	**Times**
2 times an hour	00, 30
3 times an hour	
4 times an hour	00, 15, 30, 45
5 times an hour	
6 times an hour	
10 times an hour	
12 times an hour	
15 times an hour	

Investigate what happens if the first bus comes at a different time.

Use the Maths Investigations pattern builder to check your numbers.

$5 + 15 = 20$ ✓

$20 + 15 = 35$ ✓

$35 + 15 = 50$ ✓

$50 + 15 = 65$ ✓

$10 + 30 = 40$ ✓

$40 + 30 = 70$ ✓

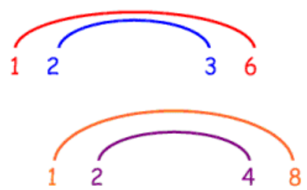
Frequency	**Times**
2 times an hour	10, 40
3 times an hour	
4 times an hour	05, 20, 35, 50
5 times an hour	
6 times an hour	
10 times an hour	
12 times an hour	
15 times an hour	

Find some real bus timetables and see if they use any of your numbers.

Name: ________________ Class: ____ Date: __________

Multiplication Square

Look at the *multiplication square* below. Work out how many factors each number has. Draw factor rainbows for the ones that have more than two factors and record your findings in a table.



Number	Factors
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
9	
10	
12	

Colour the numbers on the square. 1 has only one factor. Colour it red.

Find all the numbers with two factors. Colour them blue.

Colour the numbers with three factors orange.

Colour the numbers with four factors green.

Colour the numbers with five factors pink. And so on.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

What patterns do you get? Why?

Name: ________________ Class: ____ Date: __________

Dividing a Circle

How many seasons are there in a year?

How many months are there in a year?

How many hours are there in a day?

How many minutes are there in an hour?

60

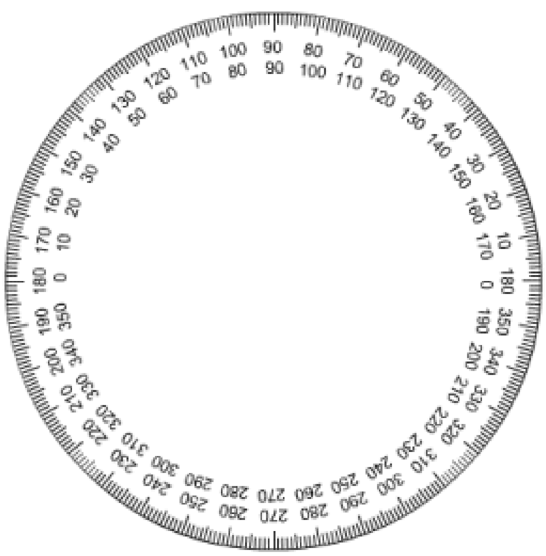
12

24

What is special about the factor rainbows for these numbers?

Why do you think ancient people chose these numbers to measure time?

Degrees are used to divide up a circle. Find a circular protractor.



How many degrees are there in a whole circle?

What is special about the factor rainbow for this number?

Why do you think ancient people chose this number to divide up a circle?