



MATHS INVESTIGATIONS SKILLS PROGRESSIONS

COUNTING, ADDITION AND SUBTRACTION

FRACTIONS, DECIMALS, PERCENTAGES & EQUIVALENCES

MULTIPLICATION, DIVISION AND FRACTIONS / % OF

Version 1: November 2015

Key Ideas in Number

Children often have difficulty with aspects of number work, particularly with missing number problems and with fractions. This is because there are gaps in their understanding.

The following progressions ensure that the key ideas are thoroughly learned and embedded. To help with planning, the concepts are grouped into **steps**. Each step corresponds roughly to one year of school, although where individual children are in this progression will vary according to their current capability.

Many of the activities require **practical equipment**. This is **essential**. **Counters** provide children with experience of quantities of number and how these relate to each other. **Metre sticks** provide a visual representation of the number line which can be used to build concepts from adding and subtracting 10 through to working with decimals and percentages.

It is also important that the children use the correct vocabulary and language structures to **talk** about the concepts they are exploring. You will need to model this for them.

There are **several progressions** which run in **parallel**.

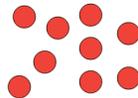
- **Counting, addition and subtraction** (beginning on Step 1)
- **What fractions are** and their **equivalences** with each other and with other things (beginning on Step 1)
- Multiplication & division and how they link with **finding fractions/percentages of numbers** and with **expressing something out of something as a fraction or percentage**. (beginning on Step 2)

In many of the sections in each progression the Maths Investigation **Owls** are referenced. The owls are carefully structured groupings of the online activities that children can work through, both in school and at home, to ensure that their understanding is reinforced.

You may wish to direct children to a particular owl at their level, but you should also allow them to dip into other owls as and when they choose. Learning is not linear! The star system in the quizzes encourages children to return to consolidate concepts learned earlier.

Essential Equipment

Counters



Metre Sticks



The photocopiable Maths Investigations activities (particularly the counting caterpillars and tap say turn cards)

iPad with Maths Investigations online activities loaded (referenced in resources column below as MI)

The four fraction lessons and pupil sheets (inserted after step 3 in the fractions progression below)

Important: Provide as much **practical work** as possible. Avoid the abstract! Also: Be a stickler for **correct speaking** of the Maths. Give children as much opportunity as possible to practise both **asking** and **answering** questions and **explaining** their thinking, using the vocabulary and language structures you have modelled for them.

Note: Activities below in red are not written yet! If you want one prioritised, please request it.

COUNTING, ADDITION AND SUBTRACTION

Step 1

Owl	Skills & Understanding	Examples
Bod 	Count in 1s Count down in 1s Odds and Evens Add/subtract 1 or 2 Number pairs that add to 10 Doubles facts	Counting within 10 $7 + 1, 8 - 1$ $4 + 2, 6 - 2$ $3 + 7 = 10$ $10 - 6 = 4$

Step 2

Owl	Skills & Understanding	Examples
Teeny 	Count in 1s beyond 10 Count in 2s Work with teens facts Number pairs that add to 20 Use so and because to link ideas Doubles facts Addition triangles	$4 + 10 = 14; ? + 10 = 17$ $16 - 10 = 6; ? - 10 = 9$ $3 + 7 = 10$ so $3 + 17 = 20$ $16 + 4 = 20$ because $6 + 4 = 10$ $6 + 6 = 12$
Didgie 	Use doubles, teens and tens pairs to work out nearby facts Add / subtract any single digits Addition of odds and evens	$6 + 7 = ?$ $9 + ? = 12,$ $13 - 9 = ?$ $15 - 7 = ?$
Mully 	Counting in tens within 100 Meaning of tens number names Number of tens in a number	$6 \text{ tens} = 6T = \text{sixty} = 60$
Honey 	Use of the metre stick to represent the number line Sequence into the hundreds Before/after/more/less Split numbers into tens and units Add / subtract 1, 2, 3 to / from a 2-digit number Tens pairs that add to 100 Inverse arrows Count / Add / subtract ten from any number Number of tens altogether	$45 + 2, 72 - 3$ $30 + 70 = 100$ $67 + 10, 49 - 10$

Step 3

Owl	Skills & Understanding	Examples
Sticky 	Count up and down in 1s, 10s & 100s to 1000 and beyond Split numbers into HTU Total number of tens and hundreds altogether in a number Add/subtract multiples of ten to / from any number Differences is distance between Add to get to the next ten	$72 + 20$, $84 - 40$ $30 + ? = 70$ $26 + ? = 30$
Fizzy 	Fives pairs that add to 100 and other tens Add/subtract five Find differences between fives Link between difference and subn	$35 + 65 = 100$ $70 - 5$ $68 + 5$
Addisub 	Add and subtract a single digit by portioning or using number bonds Choose appropriate method	$34 + 67$ $28 + ? = 51$; $51 - ? = 28$ $72 - 39$

Step 4

Owl	Skills & Understanding	Examples
Toodie 	Add a 2-digit number using partitioning Subtract by taking away or by finding differences Add or subtract using near tens	
Dessy 	Count in tenths Add subtract with tenths and whole numbers combined Make up a decimal to the next whole number	$6 + 0.5$; $8 - 0.5$; $3.5 + 0.5$, $7.5 - 0.5$ $4 - 0.1$; $3, 2 + 0.1$ etc $3.6 + ? = 4$
Multeena 	Total of hundreds in a number. Add subtract multiples of 10, 100 to/from a number	$19 \text{ hundreds} = 1900 = 1 \text{ thousand } 9 \text{ hundred}$
Stanny	The standard written method for addition and subtraction	$32 + 49$ $71 - 38$

Step 5

Owl	Skills & Understanding	Examples
Equi 	Add and subtract halves and quarters	$\frac{1}{2} + \frac{3}{4} = \text{etc}$
Decipercy 	Count in hundredths Add subtract with tenths, hundredths and whole numbers combined Add and subtract halves, fifths and tenths.	3.01, 3.02, 3.03 etc 4.26, 4.36, 4.46 etc 4.5 - 0.01; 3.62 + 0.01 0.62 + ? = 1 1 - 0.62 = ?
Thowsy	Number names beyond 10,000 Count, +/- multiples of 10, 100, 1000 into the high thousands Mental +/- strategies within 1000 using partitioning and differences Standard written methods involving more than one exchange Know when to use SWM and when to calculate mentally	265 thousands = 265,000 etc 34,265 + 7000 340 - 280 349 + 718

Step 6

Owl	Skills & Understanding	Examples
	Multiples of 1 million	
	Self-checking loops SWM decimals of different lengths	
Fradespequi 	Add and subtract fractions with denominators that are factors of 100 and their decimal equivalences	$\frac{3}{20} + \frac{2}{5}$ $0.2 - \frac{4}{25}$

Step 7

Owl	Skills & Understanding	Examples
	Add and subtract any combination of fractions and decimals using equivalences	

FRACTIONS, DECIMALS, PERCENTAGES & EQUIVALENCES

Step 1

Owl	Skills & Understanding	Examples
n/a	Half of objects: two equal pieces Write $\frac{1}{2}$ (2 on bottom - cut into 2) 'one half' = 'a half'	
n/a	Count halves Equivalences: no halves is nothing, two halves is a whole thing. Write 0 halves, 1 half, 2 halves. Top number tells how many halves. Say 1 over 2 etc.	

Step 2

Owl	Skills & Understanding	Examples
n/a	Quarters - 4 equal pieces Cut halves in half. Write $\frac{1}{4}$ (4 on bottom - cut into 4)	
n/a	Count quarters Equivalences: 0 quarters is nothing, 2 quarters is the same as a half, 4 quarters is a whole thing. Write various quarters.	
n/a	Recognise mixed halves and quarters of shapes. Denominator - what kind? Numerator - how many?	

Step 3

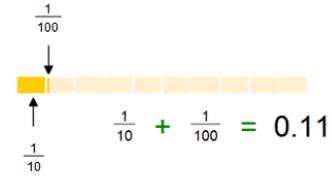
Owl	Skills & Understanding	Examples
Sticky 	Equivalence of metres and centimetres. Correct speaking of metric measurements Equivalences of 0, 1 or 2 half metres with centimetres.	1m=100cm, 2m=200cm 1m 50cm=150cm Say '1 metre 50cm', half a metre , one half of a metre , no metres . $\frac{1}{2}$ m = 50cm, 0 halves is 0cm, 2 halves is 100cm

<p>Fizzy</p> 	<p>Count first in simple halves beyond one whole and then in mixed number halves. Equivalences beyond one whole both with halves and wholes and with m and cm. Correct speaking of metric measurements</p>	<p>Count 0 halves, 1 half, 2 halves, 3 halves etc. Count 0, $\frac{1}{2}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3 2 halves = 1 whole, 4 halves = 2 wholes etc 3 halves of a m = 150cm Say 'one and a half metres'</p>
	<p>Empty fractions of shapes - ask and answer what kind? Shaded fractions - ask and answer how many?</p>	<p>Say 'thirds, fifths, sixths, eighths, tenths' Say '2 thirds, 4 fifths etc</p>
<p>Quarty</p> 	<p>Correct speaking with quarters Counting in quarters Equivalences with quarters - Equivalences between improper fraction and mixed number halves and quarters Whole and quarter metres Equivalences with cm</p>	<p>$\frac{1}{4}$ m = 25cm Say '1 quarter of' etc Count 0, a quarter, a half, 3 quarters, 1, 1 and a quarter, 1 and a half, etc. 8 quarters = 2 wholes $\frac{3}{2}$ m = $\frac{6}{4}$ m = $1\frac{1}{2}$ m = 150cm etc</p>

Step 4

Owl	Skills & Understanding	Examples
<p>Dessy</p> 	<p>Tenths of a metre (10 bits) Correct speaking Decimal notation for single place decimals Decimal - mixed number equivalences Count in 0.5s and 0.1s Equivalences: tenths, fifths, halves of a metre, decimal metres, centimetres. 1 fifth is twice as big as 1 tenth</p>	<p>Say '4 tenths of a metre' 1.5 = 1 whole and 5 bits $1\frac{5}{10} = ?$ $1\frac{1}{2} = ?$ $\frac{1}{2}$m = 50cm = 5 tenths of a metre 2.5m = $2\frac{1}{2}$m = 5 lots of 0.5m = 250cm 1 tenth of a metre = 10cm. 1 fifth of a metre = 20cm.</p>

Step 5

Owl	Skills & Understanding	Examples
Equi 	Simple fractions equivalent to one half and one quarter Simple fractions equivalent to two quarters and to three quarters Patterns in the numbers Simplify such fractions	$\frac{1}{2} = 2/4 = 3/6 =$ $\frac{1}{4} = 2/8 = 3/12 =$ $\frac{3}{4} = 6/8 = 9/12 =$ $4/16 = \frac{1}{4}$ etc
Decipercy 	Decimal notation for two place decimals - 'whole ones', 'bits' (tenths) of a metre and 'pieces' (hundredths) of a metre. Hundredths are called cm because 'cent' means 100 0.11 (eg) is made up of one tenth and one hundredth Equivalences: proper and mixed number halves and quarters with decimals, m, cm.	 $\frac{1}{4} / \frac{3}{4} \rightarrow 0.25 / 0.75$ $\frac{1}{4} / \frac{3}{4} \text{ m} \rightarrow 25\text{cm} / 75\text{cm}$ $3\frac{3}{4}\text{m} = 3.75\text{m} = 375\text{cm}$
Decipercy	Equivalence of fifths and twentieths of a metre with cm and with decimal fractions $1/20$ is half the size of $1/10$ Fractions equivalent to fifths	$1/5\text{m} = 20\text{cm} = 0.2\text{m}$ so $4/5\text{m} = 80\text{cm} = 0.8\text{m}$ $1/20\text{m} = 5\text{cm} = 0.05\text{m}$ so $7/20\text{m} = 35\text{cm} = 0.35\text{m}$
Decipercy	If you divide the numerator by the denominator of a fraction you get the decimal equivalent	$1 \div 4 = 0.25 = \frac{1}{4}$ etc. $3 \div 4 = 0.75 = \frac{3}{4}$ $2 \div 5, 3 \div 10$ etc
Decipercy	Percentages are equivalent to hundredths Convert 2dp decimals to percentages and whole number percentages to decimals	4% of the metre stick is 4cm = 0.04m. 40% of the metre stick is 40cm = 0.4m $0.6 = 60\%$, $6\% = 0.06$, $0.16 = 16\%$ etc.

Step 6

Owl	Skills & Understanding	Examples
Fradespequi 	How factors of 100 link to the equivalences between one half, one quarter, one fifth, one tenth, one twentieth, one twenty-fifth and one fiftieth and decimals. Convert fractions with these denominators to decimals and %.	$3/25\text{ths} = 0.12$ etc
Fradespequi	Express the relationship between two numbers as a fraction and then as a percentage.	4 out of 25 = $4/25 = 16\%$.
Fradespequi	Simplify fractions by dividing numerator and denominator by the same number. Change one or two place decimals and whole number percentages to fractions in their simplest form.	$0.35 = 35/100 = 7/20$
Fradespequi	Convert 2, 3 or 4 place decimals to percentages and decimal number percentages to decimals.	$0.625 = 62.5\%$, $4.9\% = 0.049$

Step 7

Owl	Skills & Understanding	Examples
	Change 2 or 3-place decimals and decimal number percentages to fractions in their simplest form.	$0.352 = 352/1000 = 44/125$
	Use SWM to convert fractions to decimals.	$5 \div 8 = 0.625$
	Change fractions with trickier denominators to decimals or % by comparing with easier ones.	40ths are ten times smaller than quarters

Step 8

Owl	Skills & Understanding	Examples
	Convert fractions to decimals and percentages where the answer is recurring.	

MULTIPLICATION, DIVISION AND FRACTIONS / % OF

Step 2

Owl	Skills & Understanding	Size of Numbers
Mully 	Multiplication using an array Multiplication sign (x) Commutativity Multiplying by 1 and 0	1, 2, 3, 4 or 5 rows of small numbers of counters. $3 \times 5 = 15$ etc
Mully	Counting in tens Meaning of tens number names	6 tens = 6T = sixty = 60
Honey 	No. of tens altogether in a number Moving of digits to left and right when \times or \div by 10 10x table facts Counting in multiples forwards and backwards - Counting caterpillar Patterns in numbers Memorise facts - Tap Say Turn Solve missing number problems	260 = 26 tens Multiples of ten from $1 \times 10 = 10$ through to eg $15 \times 10 = 150$
Honey	Factors Factor rainbows	Factors of 10 and 20

Step 3

Owl	Skills & Understanding	Examples
Divvy 	Grouping and Sharing - how many altogether? How many groups? How many in each group?	$12 \div 4 = 3$
Divvy	Doubling and halving Doubling gives even numbers Multiplying means making more Dividing means splitting up 2x table facts	Exs within 2x table: 3 is half of 6 Double 4 is 8
Divvy	Multiplication triangles, inverse arrows & fact families Position of 'altogether' number in multiplication and division 5x table facts and links with 10x	Exs within 2x, 10x, 5x tables $3 \times 5 = 15$, $5 \times 3 = 15$ $15 \div 3 = 5$, $15 \div 5 = 3$

 Sticky	Count in 100s to 1000 Multiples of 100 Doubling tens and how it links with doubling single digits	4 hundreds = 400 $60 + 60 = 120$
 Quarty	Quadruple and quarter Alternative methods Find a half, find a quarter Relate 'find a fraction' to division Count in quarters. 4x table facts	4 rows or groups of small numbers. Exs within 4x table $\frac{1}{2}$ of 16 = 8, $16 \div 2 = 8$ $\frac{1}{4}$ of 16 = 4, $16 \div 4 = 4$

Step 4

Owl	Skills & Understanding	Examples
 Dessy	Count in tenths. Understand writing of tenths as decimals.	$0.6 = 6$ tenths
 Fracto	Find $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of Find one third and one fifth of Something out of something as a simple fraction 3x table facts	Arrays of 12 and then 20 counters. $\frac{3}{4}$ of 20 = 15 3 out of 12, 8 out of 20 Exs within 3x table $\frac{1}{3}$ of 18 = 6
Fracto	Divisions that don't work exactly Express answers to divisions by 2 or 4 as remainders or fractions	Dividing 9, 11, 13 etc by 2 or 4. $\frac{1}{2}$ of 9 = 4 r1; $4\frac{1}{2}$ $\frac{1}{4}$ of 11 = 2 r3; $2\frac{3}{4}$
Fracto	Moving of digits relative to whole number place value when multiplying and dividing by 10	Two digit numbers $\times 10$ $34 \times 10 = 340$ $340 \div 10 = 34$
 Multeena	Multiply a teens number or a multiple of ten by a single digit	3, 4, 5x tables $13 \times 4 = 52$ $60 \times 4 = 240$
Multeena	The idea that eg 5 lots of 3 of anything is 15 of those things. Multiplying a multiple of 10 or 100 by 5, 4, or 3 by using tables facts and making the answer 10x bigger.	5 lots of 3 tens is 15 tens which is 150. 5 lots of 3 hundreds is 15 hundreds which is 1500. $3 \times 40 = 120$ because $3 \times 4 = 12$.

Multeena	Total no. of hundreds in a number. Multiply and divide by 10 or 100.	19 hundreds = 1900 = 1 thousand 9 hundred
Trihexnonny 	Numbers with many factors that divide up in many ways and their use in measuring Time. Links between 3, 6, 9x tables 6x, 9x table facts Finding unit fractions of Factor rainbows	6, 9x tables Factors of 12, 24, 60.
Doquadrocty 	Double and half by partitioning. Links between 2x, 4x, 8x tables 8x table facts	2-digit numbers Double 27 = 74 Half of 74 = 37
Stanny	Use the standard written method to multiply or divide by 2, 3, 4, 5 Use the inverse to solve missing number problems.	364 x 5 295 ÷ 5 etc

Step 5

Owl	Skills & Understanding	Examples
Decipercy 	The total number of tenths in a number.	2.6 = 26 tenths.
Decipercy	100%, 50%, 25%, 75% of something out of something as a simple percentage.	Numbers in 4x table 75% of 32 = 24 3 out of 12 = 25%
Decimully 	Multiply multiples of 10 and 100 (or tenths) by using tables facts and making the answer 10 or 100 times bigger (or smaller).	400 x 8 30 x 70 0.6 x 3
Decimully	Move digits relative to decimal point when x and ÷ by ten. Multiply a teens number by a multiple of ten	1 or 2 sf numbers and decimals: 40, 4, 0.4, 0.04; 360, 36, 3.6, 0.36 13 x 20
Fraperof 	Double and half 3sf numbers Find a 'fraction of' by finding the unit fraction and then multiplying.	1.75 x 2, 1370 ÷ 2 Familiar denominators (10, 2, 5, 4, 3, 6, 9, 8) of numbers in corresponding x tables. 3/8 of 56 = 7

Fraperof	Make appropriate choices to use either fraction equivalence (eg 25% of = $\frac{1}{4}$ of) or 'find 10% and then multiply', to find easy percentages such as 10%, 25%, 30%.	25% of 140 = 35 30% of 160 = 48
Fraperof	Express a remainder as a remainder or as a fraction Explore remainder patterns	Dividing by 2, 3, 4, 5 $20 \div 5$, $21 \div 5$, $22 \div 5$ SWM examples

Step 6

Owl	Skills & Understanding	Examples
Muldifrapper	Total hundredths in a number. Decimal conversion m-cm, cm-mm.	$3.35 = 335$ hundredths. $335\text{cm} = 3.35\text{m}$
Muldifrapper	Multiply or divide a 3sf whole or decimal by a single digit or by 10 or 100 using partitioning Decimal conversion with m/cm/mm	(1.65×4) $(8.25 \times 100$ etc)
Muldifrapper	Express the answer to a SWM division by 4, 5, 8 as a decimal. Explain the equivalences of answers given as decimals, fractions or remainders and explain when each of these would be appropriate. Find fractions of decimals where the answer is non-recurring.	$\frac{1}{4}$ of 259 = $259 \div 4 =$ $64\text{r}3 = 64\frac{3}{4} = 64.75$
Muldifrapper	Make appropriate choices to use either partitioning (eg 15% of = 10% of + 5% of) or 'find 1% and then multiply', to find percentages such as 1%, 3%, 15%.	10% of £2.60 = £0.26; 5% is £0.13; 15% is £0.39 3% of £24.00 is £0.24 $\times 3 = \text{£}0.72.$

Step 7

Owl	Skills & Understanding	Examples
Mulperindy 	Multiply/divide a 4sf decimal by a single digit or by 10, 100, 1000 Decimal conversion with kg/g/mg Decimal conversion with km/m/mm	2.675×3 $3.615\text{m} = 3615\text{mm}$ $0.235\text{km} = 235\text{m}$
Mulperindy	Multiplying by (eg) $\frac{1}{2}$ is the same as dividing by (eg) 2 which makes things 2x smaller; dividing by (eg) $\frac{1}{2}$ is the same as multiplying by (eg) 2 makes something 2x bigger.	Examples by $\frac{1}{2}$ or by $\frac{1}{5}$: $240 \times \frac{1}{5} = \frac{1}{5}$ of 240 $= 240 \div 5 = 48.$ $16 \div \frac{1}{5} = 16 \times 5 = 80.$
Mulperindy	Multiplying and dividing by eg 0.4 is the same as multiplying or dividing by 4 but with an answer 10 times smaller or larger	$16 \times 4 = 64$ so $16 \times 0.4 = 6.4$; $64 \div 4 = 16$ so $64 \div 0.4 = 160$
Mulperindy	Speak single digit by single digit horizontal divisions correctly and express them correctly using SWM.	5 divided by 8 = $5 \div 8$. In the SWM the number you are dividing by goes inside the box so the correct answer is 0.625.
Mulperindy	Find any percentage of any number both by dividing and multiplying and by multiplying directly by the fractional or decimal equivalent.	13% of 500: 1% of 500 = 5; $5 \times 13 = 65$ OR $0.13 \times 500 = 65.$
Mulperindy	Percentage increase and decrease	Increase £6 by 5% $5\% \text{ of } 6.00 = 0.18$ $6.00 + 0.18 = 6.18$

Step 8

Owl	Skills & Understanding	Examples
	Percentage increase and decrease using scale factors and backwards	Increase £6 by 5% $600 \times 1.05 = 618$ $618 \div 1.05 = 600$
	Divisions with recurring decimal answers.	$135 \div 6$

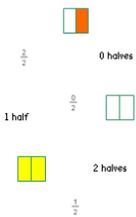


Step 1: OWL Bod

Work with Single Digits

Understanding and Skills	Activities and Resources
<p>Count in 1s</p> <p>Count down in 1s</p>	<p>Counting up and down within 10</p> <p>MI Counting in 1s to 10</p> <p>MI Counting down in 1s within 10</p>
<p>Be able to explain, with counters in arrays of two rows, how even numbers have an even (smooth) end whereas odd numbers have an odd one that sticks out.</p>	<p>$4 + 6 = 10, 5 + 3 = 8$</p> <p>MI Odds and Evens that make Ten</p> <p>MI Adding Odds and Evens</p>
<p>Add/subtract 1 or 2</p>	<p>$7 + 1, 8 - 1, 4 + 2, 6 - 2$</p> <p>MI Add 1, Add 2, Subtract 1, Subtract 2</p>
<p>Be able to explain, using counters in arrays of two rows, the number pairs that add to make ten.</p> <p>Memorise the number pairs that add to 10</p>	<p>$3 + ? = 10, 10 - ? = 4$</p> <p>MI photocopiable activity: Tap Say Turn Number Bonds</p> <p>MI Numbers that Add to Make Ten</p> <p>MI Subtracting from Ten</p>
<p>Using counters in arrays of two rows, be able to explain the doubles facts.</p> <p>Memorise the doubles facts.</p>	<p>$2 + 2 = 4, 3 + 3 = 6$ etc</p>

Step 1: Understand Halves of Objects

Understanding and Skills	Activities and Resources
<p>Understand half of a shape - cut into two equal pieces.</p>	<p>Refer to halves in everyday experience. Cutting fruit, paper etc.</p>
<p>How you write $\frac{1}{2}$. It has a two on the bottom because you have cut the shape into two equal pieces. Say one half and a half and know they are the same.</p>	<p>Have a card on the wall with $\frac{1}{2}$ and refer to it.</p> 
<p>Count halves - zero/no halves, one half, two halves. Equivalences: Understanding that zero or no halves is nothing, two halves is a whole thing.</p>	<p>Play with and count halves at meal times - biscuits, sausages, etc.</p>
<p>Write zero halves, one half and two halves. The two on the bottom tells us it is halves. The number on the top tells us how many. Say 1 over 2 to describe $\frac{1}{2}$ etc. (not 1 line 2).</p>	<p><i>See lesson 1 below.</i></p> 



Step 2: OWL Teeny

Work with Teens Numbers

Understanding and Skills	Activities and Resources
Count in 1s beyond 10 Count in 2s	MI Counting in 1s beyond 10 MI Counting in 2s
Using counters in arrays of two rows, be able to explain the doubles facts. Memorise the doubles facts. Using counters in arrays of two rows, be able to explain the teens facts. Memorise the teens facts.	Explore doubles $6 + 6 = 12$ through to $10 + 10 = 20$ MI Doubles MI Doubles subtraction facts MI photocopiable activity: Tap Say Turn Number Bonds $4 + 10 = 14$; $? + 10 = 17$; $16 - 10 = 6$; $? - 10 = 9$ MI Teens facts 1, 2, 3 MI Doubles and Teens
Using counters in arrays of two rows, be able to explain the number pairs that add to make 20 - use so and because to link ideas Addition triangles	$3 + 7 = 10$ so $3 + 17 = 20$ $16 + 4 = 20$ because $6 + 4 = 10$ MI Numbers that add to make Twenty MI Subtracting from 20 MI Addition Triangles



Step 2: OWL Didgie

Add and Subtract Single Digit Numbers

Understanding and Skills	Activities and Resources
<p>Be able to explain how, if you know your doubles, teens facts and pairs that make ten, you can work out nearby facts.</p> <p>Hence be able to add and subtract any pairs of single digits.</p>	<p>$6 + 6 = 12$ so $6 + 7 = ?$, $14 - 7 = 7$ so $15 - 7 = ?$</p> <p>MI Near Doubles</p> <p>$9 + 1 = 10$, so $9 + ? = 12$,</p> <p>MI Near Tens</p> <p>$13 - 10 = 3$ so $13 - 9 = ?$</p> <p>MI Near Teens</p> <p>MI Doubles, teens and pairs that make ten,</p> <p>MI Near Doubles, Near Tens and Near Ten Pairs</p> <p>MI Doubles, Teens and Ten Pair Subtractions</p> <p>MI Near Double, Teen and Ten Pair Subtractions</p>
<p>Be able to explain what happens when odd and even numbers are added</p>	<p>MI Odd add what equals even?</p> <p>MI Odd add what equals odd?</p>



Step 2: OWL Mully

Learn about Multiplication

Understanding and Skills	Activities and Resources
<p>Understand how multn can be investigated using rectangles.</p> <p>Given different rectangular arrays, be fluent asking and answering the three questions: 'How many rows?' 'How many in each row?' 'How many altogether?'</p> <p>Know what multiples are.</p> <p>Understand the use of the multiplication sign to mean rows of, lots of, groups of etc.</p> <p>Understand the commutative aspect. ($3 \times 4 = 4 \times 3$)</p> <p>Understand that if you multiply something by one it stays the same, if you multiply something by zero, the answer is zero.</p>	<p>Build rectangular arrays of counters (eg 3 rows of 4). Turn them round to show 4 rows of 3 and agree that this is the same. Use a variety of vocabulary to describe: rows, columns, groups, lots, sets of.</p> <p>Discuss multiples and introduce the multiplication sign.</p> <p>Explore what happens if you have just one row, or no rows, so $1 \times 4 = 4$ and $0 \times 4 = 0$.</p> <p>Use question cards with 'How many rows?' 'How many in each row?' and 'How many altogether?'</p> <p>MI: Multiplying with Rectangles and Squares</p> <p>MI: Multiplication rectangle pairs</p>
<p>Know how to count in tens. Understand eg 6 tens = 6T = 60.</p> <p>Be able to answer correctly given eg 6 tens rods 'How many tens?' (Answer 6 tens - not 6 or 60) 'How many altogether?' (Answer 60)</p> <p>Be able to explain the patterns in the 10x table.</p>	<p>Explore multiples of ten using tens rods and metre sticks. Notice that one tens rod matches with 10 on the metre stick.</p> <p>Discuss that the words for 10 and 20 are the odd ones out in the pattern - we should say 1T, 2T, 3T, but we say ten, twenty, thirty etc.</p> <p>MI: Playing with Tens</p>



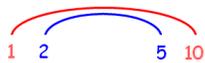
Step 2: OWL Honey

Work with Numbers up to 100

Understanding and Skills	Activities and Resources
<p>Locate numbers on line to 100 and beyond</p> <p>Count forwards and backwards from different numbers into the hundreds</p> <p>Be able to locate the teens numbers and the tens numbers and explain how they are constructed.</p> <p>Split numbers into tens and units</p> <p>Use place value to order and sequence numbers</p> <p>Use terms before/after/more/less</p>	<p>Use metre stick to locate numbers - Two hands one piano (see MI photocopiable activities)</p> <p>Count forwards and backwards in 1s and 10s.</p> <p>MI Counting in 1s into the hundreds</p> <p>MI Counting down in 1s</p> <p>MI Photocopiable activity 'Wake Up Chum'.</p> <p>Explore teens and tens (eg 4-teen = $4 + 10$ and 4-ty = 4 tens). Using the metre stick, practise counting 17, 18, 19, 20 and 70, 80, 90, 100 and check that the difference is understood.</p> <p>MI Teens and tens</p> <p>Practise partitioning numbers using arrow cards</p> <p>Practise counting collections of cubes in tens and then switching to ones: 10, 20, 30, 31, 32, 33, 34 etc.</p> <p>MI Tens and Units</p>
<p>Add / subtract 1, 2, 3 to /from a 2-digit number</p>	<p>$45 + 2$, $72 - 3$</p> <p>MI Add 1, 2 or 3, Subtract 1, 2 or 3</p>

<p>Add / subtract ten</p> <p>Be able to explain the link between addition and subtraction using inverse arrows</p>	<p>$67 + 10, 49 - 10$</p> <p>Practical work with metre stick - MI photocopiable activity Metre Stick Jumps</p> <p>MI Add Ten, Subtract Ten</p> <p>MI Inverse Arrows - Add and subtract</p>
<p>Be able to explain the tens pairs that add to 100 using metre sticks</p> <p>Be able to use 'so' and 'because' to explain the link to the number pairs that add to ten</p>	<p>Explore with the metre stick and practise speaking:</p> <p>$10 - 4 = 6$ so 10 tens - 4 tens = 6 tens so $100 - 40 = 60$</p> <p>$30 + 70 = 100$ because 3 tens + 7 tens = 10 tens because $3 + 7 = 10$</p> <p>MI Tens Pairs for 100</p>

Understand Factors of 10 and 20

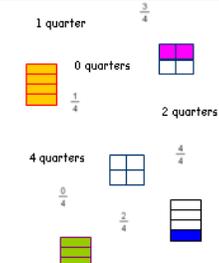
<p>Understand what factors are and know the factors of 10 and of 20.</p> <p>Eg $1 \times 10 = 10, 2 \times 5 = 10$ etc.</p>	<p>Play with 10 counters and explore how to make rectangles. Describe them using 'rows', 'in each row' and 'altogether'. Repeat with 20.</p> <p>Build the factor rainbows.</p>  <p>MI: Factors of 10 and 20.</p>
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How Tens Work beyond the 10x table

<p>Be able to say how many tens there are altogether in a multiple of ten beyond 100. (eg 260 is 26 tens.)</p>	<p>Hold ten tens rods in a 5x2 bundle and discuss how 10 tens is the same as 100. Explore what happens when you have 1 or 2 more tens - 11 tens</p>
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	<p>is 11T = 110, 12 tens is 12T = 120.</p> <p>Count in tens beyond 100.</p> <p>'How many fingers?' dialogue activity to investigate ways of working out number of fingers (and toes) in the whole class and noticing the patterns that you get with different numbers of people, eg:</p> <p>26 → 260, 27 → 270 etc</p> <p>(See MI photocopiable activities)</p> <p>MI: Playing with Tens and Numbers near 100</p>
<p>Be able to explain that when you multiply a number by 10, the digits move one place to the left, to make the number ten times bigger.</p> <p>When you divide a number by 10, the digits move one place to the right, to make the number ten times smaller.</p>	<p>Use tens rods to develop the idea that 10 tens (100) plus 3 tens (30) = 13 tens (130) and use this to explore beyond the ten times table.</p> <p>Discuss the apparent rule of 'add a zero' and introduce the idea that the zero has been inserted as a place holder because all the digits have moved one place to the right.</p> <p>Investigate that the opposite happens with division.</p> <p>MI Multiply and Divide by 10</p>
<p>Be able to count forwards and backwards in multiples of 10.</p> <p>Memorise the multiplication facts for the 10x table.</p> <p>Use the 10x table facts to solve missing number problems.</p>	<p>Play Counting stick games with 10s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn multiplication cards to learn the 10x table facts.</p> <p>(See MI photocopiable resources).</p> <p>MI: 10x table</p>

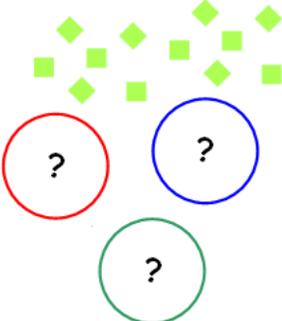
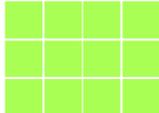
Step 2: Quarters of Objects; Halves & Quarters

Understanding and Skills	Activities and Resources
<p>Understand quarter of a shape - cut into four equal pieces. You can get quarters by cutting halves in half.</p>	<p>Referring to quarters in everyday experience. Cutting fruit, paper etc.</p>
<p>How you write $\frac{1}{4}$. It has a four on the bottom because you have cut the shape into four equal pieces.</p>	<p>Have a card with $\frac{1}{4}$ on the wall and refer to it.</p>
<p>Count quarters - zero/no quarters, one quarter, two quarters. Equivalences - Understand that zero or no quarters is nothing, two quarters is the same as a half, four quarters is a whole thing.</p>	<p>Playing with and counting quarters at meal times - biscuits, sausages, etc.</p>
<p>Write zero quarters, one quarter, two quarters, three quarters, four quarters. The four on the bottom tells us it is quarters. The number on the top tells us how many. Say 1 over 4 to describe $\frac{1}{4}$ etc.</p>	<p><i>See lesson 2 below.</i></p> 
<p>Recognise halves and quarters when they are mixed together. The number on the bottom tells us what kind of fraction it is (two for halves, four for quarters). The number on the top tells us how many.</p>	<p>Point to each fraction and say 'what kind'? (the answer should be 'halves' or 'quarters') and 'how many'? (the answer should be zero, one, two etc.)</p> <p><i>See lesson 3 below.</i></p>
<p>Introduce the terms numerator and denominator. 'Numerator': enumerate means to count - the numerator is our counting number - it tells us 'how many'. 'Denominator': means 'what kind'.</p>	<p>Point and chant aloud: 'Numerator up, denominator down, numerator up, denominator down...' etc.</p> 

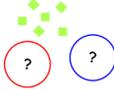
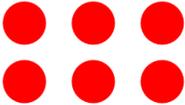


Step 3: OWL Divvy

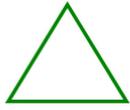
How Dividing Works - Grouping and Sharing

Understanding and Skills	Activities and Resources
<p>Understand how to divide a collection of objects into equal groups.</p>  <p>Given different groupings of objects, be fluent asking and answering the three questions: 'How many altogether?', 'How many groups?' 'How many in each group?'</p> <p>Understand the use of the division sign to mean 'shared between', 'grouped into' etc.</p> <p>Be fluent in using the words 'so' and 'because' to explain the link between division and multiplication. (eg $12 \div 3 = 4$ because $4 \times 3 = 12$) (eg $4 \times 3 = 12$ so $12 \div 3 = 4$)</p>	<p>Work with hoops and bean bags to explore what happens if you try to share eg 12 bean bags between 2 or 3 hoops. (Let the bean bags represent different creatures and the hoops represent their homes etc.)</p> <p>Explore the idea of grouping and sharing and introduce the division sign.</p> <p>See MI photocopyable activities: 'Make it fair' and 'How many each and altogether?'</p> <p>Work with rectangular arrays and explore the connection between division and multiplication.</p> <p>12 altogether, 4 in each row, 3 rows</p>  $12 \div 4 = 3$ <p>Use question cards with 'How many altogether?' 'How many rows?' and 'How many in each row?'</p> <p>MI: Multiplying and Dividing with Rectangles</p>

Doubling and Halving (Small) Numbers

Understanding and Skills	Activities and Resources
<p>Understand the terms 'doubling' and 'halving'. Explore doubling and halving numbers of things.</p>	<p>Use two hoops and some bean bags. Put eg 3 in one hoop. Put 3 in the other hoop to make 6. Discuss double 3 is 6 - half of 6 is 3, etc.</p> 
<p>Notice that whatever you double you get an even number.</p> 	<p>Recap the difference between odd and even numbers. If you put an even number of counters in two rows, they pair up. If you have an odd number there is an odd one out.</p> <p>Explore the fact that doubling will always give an even number.</p>
<p>Introduce the terms 'multiplying' by 2 and 'dividing' by 2. Multiplying means making more. Dividing means splitting up.</p> <p>Understand: lots of, times, multiply, double, half, divide & their symbols.</p> <p>Be able to explain the patterns in the 2x table. (The units digits are all even numbers. The units digits repeat after 5 stations:</p>	<p>Explore, with counters, different ways of describing the same thing. 2 lots of 3 is 6, 2 times 3 is 6, 2 multiplied by 3 is 6, $2 \times 3 = 6$, half of 6 is 3, $\frac{1}{2}$ of 6 is 3, 6 divided by 2 is 3, $6 \div 2 = 3$. <i>Lots of practice!</i></p> <p>MI: Double and half small numbers</p> <p>MI Finding Half of Even Numbers</p>
<p>Be able to count forwards and backwards in multiples of 2.</p> <p>Memorise the multiplication facts for the 2x table.</p> <p>Use the 2x table facts to solve missing number problems.</p>	<p>Play Counting stick games with 10s and 2s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn multiplication cards to learn the 2x table facts.</p> <p>(See MI photocopiable resources).</p> <p>MI: 2x table</p>

Multiplication Triangles and Inverse Arrows

Understanding and Skills	Activities and Resources
<p>Be able to use a multiplication triangle, inverse arrows and fact families to explain the link between division and multiplication.</p> <p style="text-align: center;"> $\begin{array}{ccc} & \times 5 & \\ 2 & \longleftrightarrow & 10 \\ & \div 5 & \end{array}$ </p> <p style="text-align: center;"> $2 \times 5 = 10$ </p> <p style="text-align: center;"> so </p> <p style="text-align: center;"> $10 \div 5 = 2$ </p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center; margin-right: 20px;"> 10  </div> <div style="margin-right: 20px;"> $2 \times 5 = 10$ </div> <div style="margin-right: 20px;"> $5 \times 2 = 10$ </div> <div style="margin-right: 20px;"> $10 \div 5 = 2$ </div> <div style="margin-right: 20px;"> $10 \div 2 = 5$ </div> </div> <p>(eg $10 \div 2 = 5$ because $5 \times 2 = 10$) (eg $2 \times 5 = 10$ so $10 \div 5 = 2$)</p>	<p>Explore facts families for familiar multiplication and division facts. (eg $2 \times 5 = 10$, $5 \times 2 = 10$, $10 \div 2 = 5$, $10 \div 5 = 2$.)</p> <p>Explore how inverse arrows work (see diagram opposite).</p> <p>Explore how the three numbers can be written in a triangle.</p> <p>Thoroughly explore how in multiplication the 'altogether' number comes at the end but in division you begin with it. The 'altogether number' always comes at the top of the triangle.</p> <p>Practise building fact families using cards with $\times \div =$ and appropriate triples of numbers.</p> <p>MI Multiplication Triangles</p> <p>MI Inverse Arrows - Multiply and Divide</p>

Multiplication Tables for 10 and 5

Understanding and Skills	Activities and Resources
<p>Be able to explain the patterns in the 10x and 5x tables and the links between them.</p>	<p>Explore with a metre stick how to count in tens and in fives.</p> <p>Record the 10x and 5x tables.</p> <p>Explore the patterns in the 5x</p>

	<p>table. Notice that even numbered facts end in zero, odd numbered facts end in 5. Notice that for the even numbered facts the tens digit in the answer is half the number you are multiplying by.</p> <p>Discuss why this is: if you have an even number of fives, the fives pair up to make half the number of tens; if you have an odd number of fives then there are five left over.</p> <p>Notice that the answers in the 5x table are half the answers in the 10x table, so to multiply by 5 you can multiply by ten and half it. Practise this.</p>
<p>Be able to count forwards and backwards in multiples of 5.</p> <p>Memorise the multiplication facts for the 5x table.</p> <p>Use the 5x table facts to solve missing number problems in both multiplication and division.</p>	<p>Play Counting stick games with 10s, 2s and 5s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn multiplication cards to learn the 5x table facts.</p> <p>Play Counting stick games with 10s and 2s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn multiplication cards to learn the 2x table facts. (See MI photocopyable resources).</p> <p>MI: 5x table</p> <p>MI: Divide within the 5x table</p>



Step 3: OWL Sticky

Work with Metre Sticks to Explore Numbers up to and Beyond 100

Understanding and Skills	Activities and Resources
<p>Count up and down in 1s to 1000 and beyond</p> <p>Count in 10s in the hundreds</p> <p>Split numbers into hundreds, tens and units</p>	<p>Count up and down in 1s - 458, 459, 460, 461, 462 etc,</p> <p>Count up and down in 10s - 370, 380, 390, 400, 410, 420, 430</p> <p>Practise partitioning numbers using arrow cards</p> <p>MI photocopyable activity 'Wake Up Chum'</p> <p>MI Counting in 1s into the hundreds, Counting down in 1s</p> <p>MI Counting in 10s, Counting down in 10s</p>
<p>Know how to count in hundreds up to 1000.</p> <p>Be confident discussing the total number of tens and hundreds in different numbers.</p>	<p>Explore multiples of a hundred using metre sticks. Notice that one hundred cm matches with 1 metre stick. So 200cm will be 2m etc. Recap on multiples of ten near 100 and then look at tens in multiples of 100.</p> <p>MI: Playing with Hundreds and Tens</p> <p>MI Metres and Centimetres</p>
<p>Be able to explain how to add/subtract multiples of ten to from any number on a metre stick.</p>	<p>Use the metre sticks with tens rods to explore what happens when you add or subtract multiples of ten first to/from tens numbers, then to/from fives numbers and then to/from any number.</p>

	<p>$70 + 20, 65 - 30, 84 - 40$</p> <p>MI Add Multiples of Ten</p> <p>MI Subtract Multiples of Ten</p> <p>MI Tens Doubles</p>
<p>Be able to explain, using a metre stick, how the difference between two numbers is the distance between them.</p>	<p>Using a metre stick explore the idea that the difference between two numbers is the distance between them and that you can find this by counting on or counting down.</p> <p>MI 2-digit Differences with Tens</p>
<p>Be able to explain how to get to the next ten when adding single digits within 100</p>	<p>$26 + ? = 30$</p> <p>MI Getting to the Next Ten</p>

Metres & Centimetres & Simple Halves on a Metre Stick

<p>Understand that there are 100cm in a metre, 200cm in 2 metres etc. If you have one metre and 50cm you have 150cm altogether etc.</p> <p>Know how to say mixed measurements - eg 2 metres 30 cm (we don't say the 'and').</p>	<p>Say measurements in cm and in m using the metre stick and understand the difference in size between 1cm and 1m.</p> <p>MI Metres and Centimetres</p> 
<p>Understand equivalence of half a metre with 50cm, zero halves with 0cm, 2 halves with 100cm.</p> <p>Correct speaking: half a metre, one half of a metre, no metres.</p>	<p>MI Metres and Centimetres with Simple Halves</p>

How hundreds work

<p>Know how to count in hundreds up to 1000.</p>	<p>Explore multiples of a hundred using metre sticks. Notice that one hundred cm matches with 1 metre stick. So 200cm will be 2m etc.</p> <p>MI: Playing with Hundreds and Tens</p> <p>MI Metres and Centimetres</p>
<p>Be able to explain the links between doubling and halving single digits and multiples of ten.</p>	<p>Explore how eg $6 + 6 = 12$, so 6 tens + 6 tens = 12 tens, so $60 + 60 = 120$ etc</p> <p>MI Tens Doubles</p> <p>Explore the the different meanings of x and divide etc and discussing the relationship between</p> <p>MI Double and half multiples of ten</p>



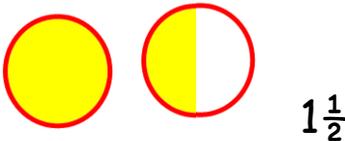
Step 3: OWL Fizzy

Work with Fives

Understanding and Skills	Activities and Resources
<p>Be able to explain the fives pairs that add to 100, eg in $35 + 65 = 100$, 3 tens + 6 tens makes 9 tens and the extra units add to make an extra ten which makes 100.</p> <p>Be able to explain how to add a variety of fives pairs together to make tens</p>	<p>Use metre sticks, tens rods and unit cubes to investigate fives pairs that make 100. Explore the different ways of understanding how it works:</p> <p>Eg 3 tens + 6 tens = 9 tens and the extra 10 units make 100</p> <p>Eg $30 + 70 = 100$. 35 is five more than 30, 65 is five less than 70, so $35 + 65 = 100$.</p> <p>MI Tens and Fives Pairs for 100</p> <p>MI Add Fives Pairs to Make Tens</p>
<p>Be able to explain how to find the ending number, starting number and action number in problems where you add/subtract 5 to/from a larger number</p>	<p>Work with floor number tracks. First explore adding and subtracting 5.</p> <p>$70 - 5 = 65$, $? - 5 = 52$</p> <p>$68 + 5$, $? + 5 = 32$</p> <p>MI Adding 5 to / Subtracting 5 from Zeroes and Fives</p> <p>MI Adding 5 to / Subtracting 5 from 1s and 6s / 2s and 7s</p>
<p>Be able to explain using a metre stick how to find the difference between two multiples of five.</p>	<p>Reinforce the idea that the difference between two numbers is the distance between them on a metre stick and that you can find this by counting/jumping on or</p>

	<p>counting/jumping down.</p> <p>MI 2-digit Differences with Fives</p>
<p>Be able to explain, using a metre stick, the link between difference and subtraction.</p>	<p>Explore the idea that you can find the difference between eg 60 and 40 ($60 - ? = 40$) by starting with 60cm and chopping the 40cm off the beginning. This gives the same answer as when you subtract 40 from 60 ($60 - 40 = ?$) by chopping 40cm off the end.</p> <p>MI The Link between Difference and Subtraction</p>

Metres & Centimetres & Mixed Number Halves on a Metre Stick

Understanding and Skills	Activities and Resources
<p>Explore what happens when you get more than 2 halves. Count halves beyond one whole - 2 halves, 3 halves, etc. Equivalences - 2 halves is the same as 1 whole, 4 halves is 2 wholes etc.</p>	<p>Count halves of objects.</p>
<p>An alternative way of counting in wholes and halves. Zero, a half, one, one and a half, two, two and a half, three etc.</p> <p>Correct way of saying 'half a metre' and 'one and a half metres' - where the 'and' comes; the plural of metre.</p>	<p>Count wholes and halves of objects. Match mixed number halves with diagrams.</p> <div style="text-align: center;">  </div>
<p>Count whole and half metres.</p> <p>Explore equivalences with centimetres.</p>	<p>Discuss whole and half metres. Overlap metre sticks to illustrate.</p> <p>MI Metres and Centimetres with Mixed Number Halves.</p>



Step 3: OWL Addisub

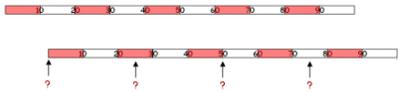
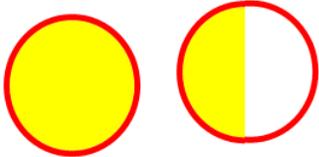
Add / Subtract Single Digits to/from Other Numbers

Understanding and Skills	Activities and Resources
<p>Be able to explain the number pairs that add to 100, eg in $36 + 64 = 100$, 3 tens + 6 tens makes 9 tens and the extra ten makes 100.</p>	<p>MI Number pairs for 100</p>
<p>Be able to explain different methods for adding/subtracting a single digit to/from a larger number.</p> <p>Give appropriate reasons for choosing a particular method</p> <p>Investigate the patterns that emerge through repeated addition and subtraction of single digits.</p>	<p>Explore the two common methods of +/- a single digit to / from another number. (a) jumping to the next ten and +/- the rest (eg $38 + 2 = 42$). (b) using number bonds (eg $8 + 7 = 15$ so $38 + 7 = 45$). Discuss when each is better.</p> <p>MI Add/Subtract a Single Digit by Jumping on/back; Add a Single Digit using Number Bonds; Subtract a Single digit by Adding</p> <p>Use the express elevator to explore how to add or subtract 9 or 8 by first + or - 10 and then going back 1 or 2 the other way.</p> <p>MI Quick Ways of Adding / Subtracting 9 (and 8)</p> <p>Patterns adding different digits</p> <p>MI Add 8 to Even Numbers, Subtract 8 from Even Numbers</p> <p>MI Photocopiable Activity - Number Chains and Digit Wheels</p>



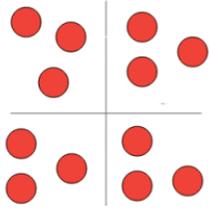
Step 3: OWL Quarty

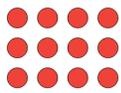
Metres & Centimetres, Halves & Quarters on a Metre Stick

Understanding and Skills	Activities and Resources
<p>Understand equivalence of 1 quarter of a metre with 25cm, zero quarters with 0cm, 2 quarters with a half with 50cm, $\frac{3}{4}$m with 75cm, four quarters with 100cm.</p> <p>Correct speaking: 1 quarter of, a quarter of, 3 quarters of a metre.</p>	<p>MI Metres and Centimetres with Quarters.</p> <p>MI Metres and Centimetres with Halves and Quarters.</p> 
<p>Alternative way of counting in wholes, halves and quarters. Zero, a quarter, a half, three quarters, one, one and a quarter, one and a half, etc.</p> <p>Correct way of saying 'one and a half'- where the 'and' comes.</p>	<p>Count wholes, halves and quarters of objects. Match mixed number quarters with diagrams.</p>
<p>Explore what happens when you get more than 2 halves. Count halves beyond one whole - three halves, four halves, etc. Equivalences - Understand that two halves is the same as one whole, four halves is the same as two wholes etc.</p>	<p>Count halves of objects.</p> 
<p>Understand equivalences between improper fraction and mixed number halves. (three halves = one and a half etc).</p> <p>Count whole and half metres. Explore equivalences with centimetres.</p>	<p>Play with halves of objects.</p> <p>Discuss halves of metres.</p> <p>MI Metres and Centimetres with Two Kinds of Halves.</p> <p>MI Counting with Halves</p>

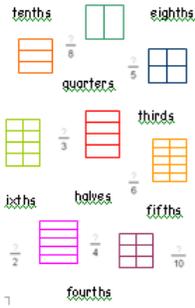
<p>Explore what happens when you get more than 4 quarters. Count quarters beyond one whole - five quarters, six quarters, etc. Equivalences - Understand that four quarters is the same as one whole, eight quarters is the same as two wholes etc.</p>	<p>Count quarters of objects. Explore what $\frac{1}{2} + \frac{1}{4}$ is. MI Counting in Quarters (to include what is 1/2 plus a 1/4)</p>
<p>Understand equivalences between improper fraction and mixed number halves and quarters. (three halves = one and a half = six quarters = one and two quarters etc). Count whole and quarter metres. Explore equivalences with centimetres.</p>	<p>Play with halves and quarters of objects. Discuss halves and quarters of metres. MI Metres and Centimetres with Many Quarters.</p>

Quadruple and Quarter Numbers

Understanding and Skills	Activities and Resources
<p>Understand the term 'quadrupling' and 'quartering'. Explore quadrupling and quartering numbers of things.</p>	<p>Use four hoops and some bean bags. Put eg 3 in one hoop. Put 3 in each of the other hoops to make 12. Discuss quadruple 3 is 12 - quarter of 12 is 3, etc.</p>
<p>Be able to explain the two different ways of quadrupling and quartering. First double and double again, half and half again. Then multiplying and dividing by 4. Understand that finding a quarter of something is the same as dividing by 4. Be able to find a quarter/half by</p>	<p>Discuss equal numbers of counters in four quadrants. (a grid divided into two boxes).</p>  <p>Discuss counters in arrays with</p>

<p>sharing/grouping/dividing.</p> <p>Relate halves and quarters to 2 and 4 times tables.</p>	<p>four rows.</p>  <p>MI Quartering Numbers in the 4x Table</p>
<p>Discuss 'multiplying' by 4 and 'dividing' by 4. Revise multiplying means making more. Dividing means splitting up.</p> <p>Understand: lots of, times, multiply, quadruple, quarter, divide and their symbols.</p>	<p>Explore, with counters, different ways of describing the same thing. 4 lots of 3 is 12, 4 times 3 is 12, 4 multiplied by 3 is 12, $4 \times 3 = 12$, quarter of 12 is 3, $\frac{1}{4}$ of 12 is 3, 12 divided by 4 is 3, $12 \div 4 = 3$.</p> <p><i>Lots of practice!</i></p>
<p>Count in quarters.</p>	
<p>Be able to count forwards and backwards in multiples of 4.</p> <p>Memorise the multiplication facts for the 4x table.</p> <p>Use the 4x table facts to solve missing number problems.</p>	<p>Play Counting stick games with 10s, 2s, 5s and 4s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn multiplication cards to learn the 4x table facts. (See MI photocopiable resources).</p> <p>MI 4x table</p> <p>MI Divide within the 4x table.</p>

Step 3: Thirds, Fifths, Sixths, Eighths, Tenths of Objects

Understanding and Skills	Activities and Resources
<p>Understand common fractions of shapes - thirds, fifths, sixths, eighths, tenths.</p> <p>Cutting something into sixths means cutting it into six equal pieces, etc.</p> 	<p>Discuss the words - the idea that halves and quarters are the weird names - others fit with the words - thirds (3 bits), sixths (6 bits), etc - halves should be twoths(!), quarters are sometimes called fourths.</p> <p>Distinguish between diagrams where some cuts are equal and some are not.</p>
<p>Work out the amount of a shape indicated (simple fractions).</p> <p>Revise the terms numerator and denominator. 'Numerator': enumerate means to count - the numerator is our counting number - it tells us 'how many'. 'Denominator': means 'what kind'.</p>	<p>Empty fraction diagrams and numbers with only denominator filled in. Say: what kind - point down. <i>See Lesson 4 below.</i></p> <p>Fraction diagrams with parts shaded. Say: what kind - point down, how many - point up.</p>

Four lessons & pupil sheets for fractions (Steps 1-3)

(Step 4 skills continue on page 13 below.)

Lesson 1: Halves

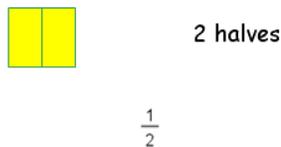
Teaching:



When you cut a shape two equal pieces, each piece is called a **half**.



One half is written $\frac{1}{2}$. You read this as **1 over 2**.



Two halves is written $\frac{2}{2}$. You read this as **2 over 2**.

Ditto for no halves or zero halves.

Activity: Leader (child or teacher) points to *anything* on diagram. Group chorus the response.

One half, no halves, two halves.

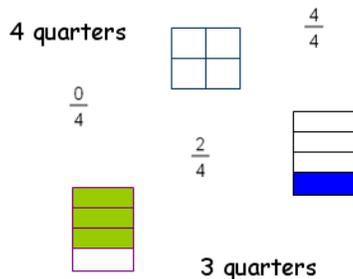
Repeat with just the numbers: **1 over 2, 2 over 2, 0 over 2.**

Lesson 2: Quarters



Teaching:

When you cut a shape four equal pieces, each piece is called a **quarter**.



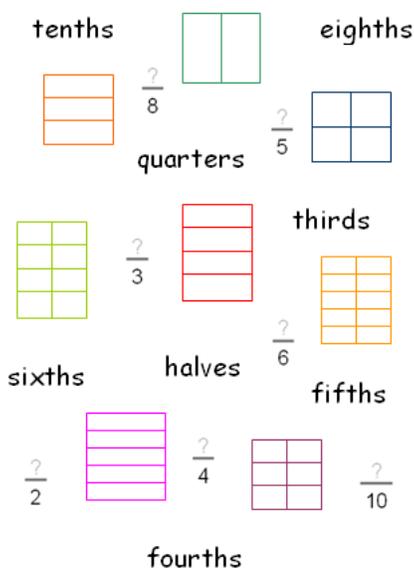
Activity: as above.

Activity C: Leader (child or teacher) points to *anything* on diagram and asks the question: What kind? Answer as above. Then asks the follow-up question: **How many?**

Group chorus the response.

One half, no halves, two halves, one quarter, no quarters, three quarters etc.

Lesson 4: Empty fraction diagrams



Teaching:

Fractions are when you cut shapes into **equal** parts.

A fraction is written with two numbers one above the other.

The **bottom number** tells us **what kind** of fraction we have.

For **thirds** we use a three because we have chopped the thing into **three equal pieces**, etc.

This bottom number is called the **denominator**.

The **top number** tells us **how many** of the fraction we have.

This top number is called the **numerator**. In these diagrams we have left the numerator as a question mark.

Activity: Leader (child or teacher) points to *anything* on diagram and asks the question: What kind?

Group chorus the response.

Halves, Thirds, Quarters, Fifths etc.

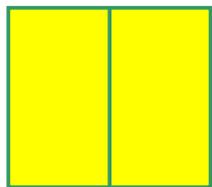
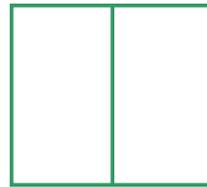


$$\frac{2}{2}$$

0 halves

$$\frac{0}{2}$$

1 half



2 halves

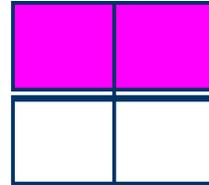
$$\frac{1}{2}$$

1 quarter

$$\frac{3}{4}$$



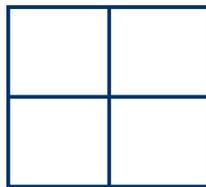
0 quarters



$$\frac{1}{4}$$

2 quarters

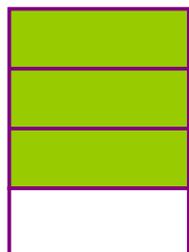
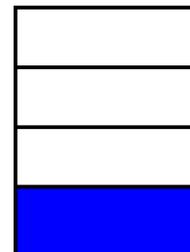
4 quarters



$$\frac{4}{4}$$

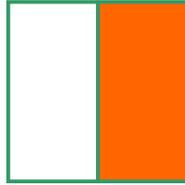
$$\frac{0}{4}$$

$$\frac{2}{4}$$



3 quarters

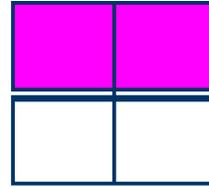
1 quarter



$$\frac{3}{4}$$



0 quarters



$$\frac{1}{4}$$

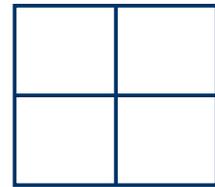


$$\frac{0}{2}$$

2 quarters

4 quarters

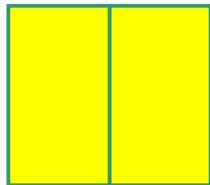
$$\frac{4}{4}$$



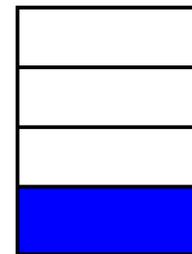
$$\frac{0}{4}$$

0 halves

2 halves



$$\frac{1}{2}$$



1 half

$$\frac{2}{4}$$



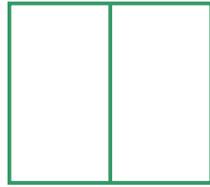
$$\frac{2}{2}$$

3 quarters

tenths

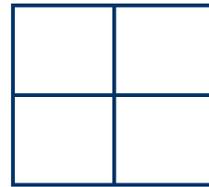


$$\frac{?}{8}$$



eighths

$$\frac{?}{5}$$



quarters

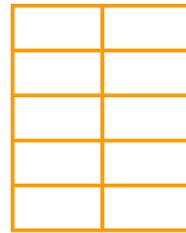


$$\frac{?}{3}$$



thirds

$$\frac{?}{6}$$

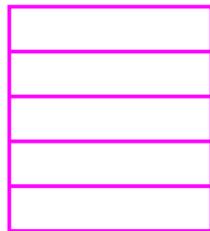


sixths

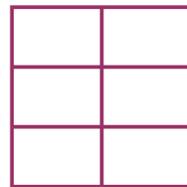
halves

fifths

$$\frac{?}{2}$$



$$\frac{?}{4}$$



$$\frac{?}{10}$$

fourths



Step 4: OWL Toodie

Add & Subtract 2-digit Numbers

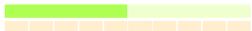
Understanding and Skills	Activities and Resources
Explain how to add a variety of 2-digit numbers together to make tens	$42 + 38 = ?$ MI Adding to make tens
Be able to use a metre stick to explain how to subtract a 2-digit number from a multiple of ten by first subtracting the tens and then the units. Record using arrow diagrams.	Use a metre stick, tens rods and unit cubes to explore eg $70 - 21$, $70 - 22$ etc, then $50 - 15$, $50 - 16$ etc. MI Take away 2-digit Numbers from Tens
Be able to explain how to use the difference method to subtract a 2-digit number from a multiple of ten.	MI Subtracting from Tens using Differences $36 + ? = 50$ so $50 - 36 = ?$ etc
Be able to explain how to add mentally any 2-digit number to a larger number using the partitioning method	Use the partitioning method to add any 2-digit numbers together. Eg: $ \begin{array}{r} 37 + 28 \\ \downarrow \quad \downarrow \\ 50 + 15 \\ \downarrow \quad \downarrow \\ 65 \end{array} $ Learn to record working this way. Solve addition problems in context. MI Adding by Partitioning
Be able to explain how to subtract mentally a 2-digit number to/from a larger number by first subtracting the tens and then the units	MI Subtract 2- digit numbers by Taking Away Solve subtraction probs in context.

<p>Be able to explain how to solve mentally any 2-digit number subtraction problem by finding the difference between them.</p>	<p>Revise the idea that finding the difference between two numbers gives you the same answer as 'taking away'.</p> <p>Show that if you use the difference method you only need two jumps - to the next ten and then on to the actual number.</p> <p>Eg to work out $82 - 38$: Difference between 82 and 38:</p> <p>$38 + \text{what} = 40$</p> <p>$40 + \text{what} = 82$</p> <p>Record using arrows.</p> <p>38 $\xrightarrow{+2}$ 40 $\xrightarrow{+42}$ 82</p> <p>Answer: 44</p> <p>MI 2-digit Subtracting using Differences</p>
<p>Be able to add or subtract $9/8$, $19/18$, $29/28$, $39/38$ etc by using the nearest ten and then adding or subtracting 1</p>	<p>Add and Subtract near Tens</p>
<p>Be able to explain how to solve mentally any 2-digit number subtraction problem by finding the difference between them, using near tens or by taking away tens and then units.</p> <p>Be able to explain why a particular method was chosen.</p>	<p>Explain that sometimes the easiest way to 'do a subtraction' is to take away. Other times it is easiest to find the difference between the two numbers etc.</p> <p>Explore which strategy is best for different examples.</p>



Step 4: OWL Dessy

Single Place Decimals & Equivalences with Fractions ($\frac{1}{2}$ s, 10ths, 5ths)

Understanding and Skills	Activities and Resources
<p>The idea of fractional tenths of a metre - called tenths not because they stop at the tens but because there are ten of them in 100cm.</p>	 $\frac{1}{2} = \frac{?}{10} \quad \frac{1}{2} = ?$ $0.5 = \frac{?}{10}$
<p>Understand that you say one metre, half a metre, three tenths of a metre, no metres etc.</p>	
<p>Know how to count in tenths up to one whole. Understand eg 6 tenths = 0.6.</p> <p>Understand equivalence of half a metre with 50cm and with 5 tenths of a metre.</p>	<p>Explore tenths of a metre using a metre stick. Notice that one tenth of a metre matches with 10cm on the metre stick.</p> <p>MI Metres and Centimetres with Halves and Tenths</p>
<p>Understand decimal notation for single place decimals - 'whole ones' and 'bits' (tenths) of a metre.</p> <p>Understand that 1.5 can also be written as a mixed number: 1 and 5 10ths.</p>	<p>Using the metre stick, introduce the idea of coloured 'bits' of a metre. Establish that there are ten of them. Agree that they are called tenths. Explore the decimal notation.</p> <p>MI How do Decimals Work? - Part 1</p>
<p>Count in 0.5s and 0.1s.</p> <p>Understand equivalence between eg 2.5m and 2$\frac{1}{2}$m and 5 lots of 0.5m and 250cm.</p>	<p>MI Decimal Halves</p> <p>MI Decimal Tenths and Halves</p> <p>MI Metres and Centimetres with Many Tenths</p>
<p>Understand equivalences between tenths, fifths and halves of a metre with decimal fractions of a</p>	<p>Explore with blu tac where to chop a metre stick to divide it into 2, 10,</p>

<p>metre and with centimetres.</p> <p>Understand that one fifth is twice as big as one tenth. One tenth of a metre = 10cm. One fifth of a metre = 20cm.</p> <p>Count in 0.2s.</p>	<p>5 equal pieces.</p> <p>MI Metres and Centimetres with Tenths and Fifths</p> <p>MI Decimal Tenths and Fifths.</p> <p>MI Counting in Decimal Tenths, Halves and Fifths</p>
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Add & Subtract with Simple Decimals

<p>Count in 0.5s and 0.1s.</p> <p>Add/subtract 0.5 to/from whole numbers and decimal halves</p> <p>Add subtract 0.1 to/from whole numbers and single place decimals</p>	<p>$6 + 0.5$; $8 - 0.5$; $3.5 + 0.5$, $7.5 - 0.5$</p> <p>MI Add and Subtract 0.5</p> <p>$4 - 0.1$; $3.2 + 0.1$ etc</p> <p>MI Add and Subtract 0.1</p>
<p>Be able to explain what you need to add to a single place decimal to get to the next whole number</p>	<p>$3.6 + ? = 4$</p> <p>MI 'Getting to the next whole'</p>

Add & Subtract Multiples of 10, 100 to/from a number

<p>Mentally add and subtract 10, 100, 1000 to/from a number.</p> <p>Mentally add and subtract multiples of 10, 100 to/from a number.</p>	<p>MI Add, subtract 10, 100, 1000</p>
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Step 4: OWL Fracto

Find Easy Unit Fractions Using Multiplication Facts

Understanding and Skills	Activities and Resources
<p>Be able to explain how to find $\frac{1}{2}$, quarters, thirds fifths and tenths of small numbers.</p>	<p>Use arrays of 12 counters to recap on finding half of numbers and then quarters. Extend to counting quarters: one quarter of 12 is 3, so two quarters of 12 will be 6, etc. Introduce and discuss thirds of 12. Repeat with 20 counters - fifths.</p> <p>MI Fractions of 12. Fractions of 20</p> <p>MI Find Unit Fractions Using Multiplication Facts; Find Fifths & Tenths using Tables</p>
<p>Understand the concept of 'something out of something' and be able to express this as a simple fraction.</p>	<p>Explore eg numbers out of 12 or out of 10 and 20 (6 out of 12, 3 out of 12, 8 out of 12 etc) and what these would be as fractions.</p> <p>MI Something out of twelve as a fraction; Something out of 10 or 20 as a fraction</p>
<p>Be able to count forwards and backwards in multiples of 3.</p> <p>Memorise the multiplication facts for the 3x table.</p> <p>Use the 3x table facts to solve missing number problems.</p>	<p>Play Counting stick games with 10s, 2s, 5s, 4s and 3s. Play Thigh Clap Snap Snap. Use the counting caterpillar and Tap Say Turn cards to learn the 3x table facts.</p> <p>(MI photocopiable resources)</p> <p>MI 3x table; Divide within the 3x table</p>

Find Halves of Odd Numbers and Quarters with Remainders

Understanding and Skills	Activities and Resources
<p>Be able to explain what happens when you try to half an odd number or quarter a number which is not a multiple of four.</p> <p>Be able to express answers to divisions as remainders or fractions.</p>	<p>Use 7 and 9 counters to explore halving and then 9, 10, and 11 counters explore quartering numbers that don't divide nicely.</p> <p>Explore the different ways of recording the question ($9 \div 4$, $\frac{1}{4}$ of 9) and the answer using remainders (2r1) and mixed numbers ($2\frac{1}{4}$).</p> <p>MI Finding Half of Odd Numbers</p> <p>MI Finding Quarters with Remainders</p>

Multiply and Divide by Ten

<p>Using the vocabulary 'ten times bigger' and 'ten times smaller', be able to explain the moving of digits relative to whole number place value when multiplying and dividing by 10</p>	<p>Through building number patterns, explore the idea that since 12 tens = 120 then $12 \times 10 = 120$ etc. Extend to division as the inverse. $180 \div 10 = 18$ etc.</p> <p>MI Multiply and Divide by 10</p>
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Step 4: OWL Multeena

Multiply and Divide by a Hundred with Whole Numbers

<p>Know how to count in tens and hundreds beyond 1000.</p> <p>Understand eg 19 hundreds = 1900 = 1 thousand 9 hundred.</p> <p>Be able to explain the patterns in the 100x table.</p> <p>Be able to multiply and divide a variety of numbers by 10 or 100.</p>	<p>Recap multiples of a hundred using metre sticks. Notice that one hundred cm matches with 1 metre stick. So 200cm will be 2m etc.</p> <p>Explore hundreds beyond 1000. Eleven hundreds = 1100 etc.</p> <p>Discuss names of years and notice the patterns that you get, eg:</p> <p>1900 → 19 hundred → one thousand nine hundred</p> <p>Discuss the fact that if you multiply/divide by a hundred it looks like you 'add/subtract two zeroes'. What is actually happening is that the number is a hundred times bigger/smaller, so the digits are moving two places to the left/right in the HTU columns. Compare this with the effect when multiplying or dividing by 10.</p> <p>MI: Playing with Hundreds and Numbers near 1000</p> <p>MI: Multiply and Divide by 100</p> <p>MI Hundreds and Tens Doubles</p>
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Multiply teens and tens by single digits in 3, 4, 5x tables

Understanding and Skills	Activities and Resources
<p>Use so to explain how to multiply a teens number such as 16 by 5, 4, or 3, by first finding ten lots, then finding 6 lots and then adding them together.</p> <p>(eg 10 lots of 4 = 40 and 6 lots of 4 = 24 so 16 lots of 4 = 40 + 24 = 64.)</p>	<p>MI Multiply Teens Numbers by 5</p> <p>MI Multiply Teens Numbers by 4</p> <p>MI Multiply Teens Numbers by 3</p>
<p>Understand that eg 5 lots of 3 of anything is 15 of those things.</p> <p>Be able to explain that if you are counting tens the answer will be ten times bigger, if you are counting hundreds the answer will be a hundred times bigger.</p> <p>Use so or because to explain how to multiply a multiple of 10 or 100 by 5, 4, or 3 by using tables facts and making the answer ten times bigger.</p> <p>(eg $3 \times 5 = 15$ so $30 \times 5 = 150$ so $300 \times 5 = 1500$; $30 \times 5 = 150$ because $3 \times 5 = 15$)</p>	<p>Explore using unit cubes, tens rods and hundreds flats the idea that 5 lots of 3 units is 15 units. 5 lots of 3 tens is 15 tens which is 150. 5 lots of 3 hundreds is 15 hundreds which is 1500.</p> <p>Repeat with 4 lots of 3, 30 and 300.</p> <p>MI Multiply and Divide with Tens by 5</p> <p>MI Multiply and Divide beyond the 5x table</p> <p>MI Multiply and Divide with Tens by 4</p> <p>MI Multiply and Divide with Tens by 3</p> <p>MI 20x table</p> <p>MI Multiply Single Digits by Tens</p>



Step 4: OWL Trihexnonny

Multiplication Tables for 3, 6, 9 and factors of 12, 24 & 60

Understanding and Skills	Activities and Resources
<p>Understand the links between the 3, 6 and 9 times tables.</p> <p>Be able to explain the patterns in the 9x table.</p> <p>Be able to multiply a teens number or multiple of 10 by 3, 6, or 9.</p>	<p>MI Multiply and Divide beyond the 3x table</p> <p>MI 6x table, Multiply and divide beyond the 6x table.</p> <p>MI 9x table, 9x table patterns</p>
<p>Be able to explain the link between dividing by 3, 4, 5, 6 and finding a third, a quarter, a fifth and a sixth of a number.</p> <p>Be able to explain what a factor is and draw a factor rainbow.</p>	<p>Investigate the factors of 12, 24, or 60 as appropriate to level. Build factor rainbows.</p> <p>MI: Factors of 12, 24 & 60</p>
<p>Know why time is segmented in 12, 24, 60.</p> <p>Be able to explain equivalences 1 hour = 60 minutes, $\frac{1}{2}$ hour = 30 minutes, $\frac{1}{4}$ hour = 15 minutes, $\frac{3}{4}$ hour = 45 minutes.</p>	<p>Discuss the meaning of halves and quarters with reference to the hour. Investigate $1\frac{1}{2}$ hours, $2\frac{1}{4}$ hours etc.</p> <p>Practise counting in quarters. ($0\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ 1 $1\frac{1}{4}$ $1\frac{1}{2}$ etc)</p> <p>Discuss ancient people watching the skies and noticing the seasons. About 13 moonths in a year. But they divided year into 12 months instead because 12 can be divided nicely into 2, 3 or 4 equal parts. 24 hours and 60 minutes used likewise.</p> <p>MI Hours and minutes with halves and quarters</p>



Step 4: OWL Doquadrocty

Double and half 2-digit numbers

Understanding and Skills	Activities and Resources
<p>Understand that halving (dividing by 2) is the opposite of doubling (multiplying by 2).</p> <p>Double & half two digit numbers.</p>	<p>Partition numbers into tens and units. Idea that to double the number you can double the tens, double the units and add them together. Ditto for halving.</p> <p>Whole class activity: Upstairs Downstairs (see MI photocopiable resources)</p> <p>Explore what happens when you half a multiple of ten (you get a number ending in 5). Compare with halving single digit odd numbers and getting halves.</p> <p>Eg half of 70 = 35, half of 7 = $3\frac{1}{2}$.</p> <p>MI (as appropriate to level): Double and Half small numbers, Double and half multiples of ten, Double tens and hundreds, Double multiples of five, Double 2-digit numbers, find halves of numbers to 100.</p>
<p>Understand the links between the 2, 4 and 8 times tables.</p> <p>Be able to explain the patterns in the 8x table.</p> <p>Be able to multiply a teens number or multiple of 10 by 2, 4, or 8.</p>	<p>MI Multiply and Divide beyond the 4x table</p> <p>MI 8x table</p> <p>MI Multiply and divide beyond the 8x table.</p>

[